Competition For vs. On the Rails: A Laboratory Experiment*

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Abstract

Several European countries and Japan are in various stages of privatizing and/or introducing more competition in passenger rail service. This process has been furthered by a directive from the Commission of the European Communities (1991) requiring member states to separate operations from infrastructure on the books and give international groupings of trains access to their infrastructure. In the Netherlands, the Ministry of Transport, Public Works, and Water Management was assigned responsibility for making a recommendation to Parliament for choosing between competition for the rails and competition on the rails in increasing competition in the supply of passenger rail service. The Ministry commissioned the experiments reported here in order to acquire better understanding of the properties of the two alternative types of competition in the context of a simple stylized rail network. The experimental rail network includes station complementarity and time slot substitutability. It also includes tradeoffs between local and express trains. Competition on the rails involves allocation of rights to use station and time slot routes by price bids in a combinatorial auction. Competition for the rails involves allocation of rights to regional monopolies by fare-structure bids for supplying a pre-specified minimum transport schedule. The experiments include both allocation of rights and scheduling of trains on the network. The two forms of competition are evaluated with various criteria developed by the Ministry, including market prices and allocative efficiency. The experimental data support the conclusion that competition for the rails gives superior results, but only if the pre-specified minimum transport schedule is not too inefficient.

Keywords: experiment, privatization, rail network, combinatorial auction.

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1. Introduction

Several European countries and Japan are in various stages of privatization of passenger rail services and/or the introduction of more competition in the market for these services. In many countries, these services were formerly supplied by vertically integrated state monopolies or strongly regulated private monopolies. For example, in the Netherlands, the passenger service has been in the hands of a monopoly (“De Nederlandse Spoorwegen,” or “NS”) since 1937. This monopoly involved both the infrastructure and its various uses. It was created in order to better coordinate the rail services which had previously been run by small companies, each with a limited number of routes. In order to keep control over this monopoly, the Dutch government became the sole shareholder and introduced severe regulation. The NS became very dependent on the government for financial support and was forced to adhere to strict government policy, both with respect to routing and pricing.

A common pattern in Europe over the past decade has been to adopt partial privatization in which a state monopoly retains ownership of the infrastructure while private firm(s) acquire rights to use it. This process has been furthered by a directive from the Commission of the European Communities (1991) requiring member states to separate operations from infrastructure on the books and give international groupings of trains access to their infrastructure. The idea behind this separation is that the infrastructure might involve decreasing average costs and hence natural monopoly arguments for government intervention might be valid. In contrast, operation of rail transportation services does not ordinarily involve decreasing average costs.

The course of partial privatization has varied between countries and over time within countries. In the Netherlands, the first stage was to formally end NS’s responsibility for development and maintenance of the infrastructure.\(^1\) NS retained the rights to use the infrastructure (hereafter, the “rails”) as a regulated private monopoly, however regulation (and subsidization) have been decreasing since 1991.\(^2\) In this way, a deregulated private monopoly is being created. The next stage will involve the introduction of competition. Two possibilities are currently under consideration: either competition for the rights to regional monopolies or competition on location and time slot routes in the network; that is, either competition for or competition on the rails. The Netherlands Ministry of Transport, Public Works, and Water Management (hereafter, the Ministry) was assigned responsibility for making a recommendation to the Netherlands Parliament for choosing between competition for and on the rails.

\(^1\)In fact, the NS still participates in the government organization now in charge of the infrastructure. However this participation is expected to end before the year 2000.
\(^2\)The subsidies will be down to zero by the year 2000. In that year, the government will start to charge the NS a fee for use of the infrastructure.
The Ministry commissioned the experiments reported in this paper in order to acquire better understanding of the properties of the two alternative types of competition in the context of a simple stylized rail network.

When commissioning the experiments, the Ministry had certain ideas with respect to what the mechanisms concerning competition for and on the rails would look like. These ideas determined the structure of our experiments to a large extent. Here, we give a brief overview of these ideas. More details are given below. In competition for the rails, the railway network would be split in regions. Operators would bid in an auction for the monopoly rights to a complete region for a limited amount of time. The government would provide a minimum schedule that would have to be run in a region. In the auction used for allocating rights, operators would bid the passenger ticket prices (or fares) to be charged. Those willing to charge the lowest prices would be given the rights. The prices would have to be charged for all transport, not just on the minimum schedule. In competition on the rails, the government would distinguish a large number of individual route/time slot combinations (e.g., Rotterdam-Amsterdam at 7:10 a.m.). The rights to each route/time slot would be allocated in a simultaneous auction. Revenue would go to the government and there would be no minimum schedule.

Note that there are important differences between these two plans for introducing competition. The most important differences are in the procedures for allocating rights and in the minimum schedule that is used in competition for the rails but not in competition on the rails. These differences must be dealt with in a way that makes a straightforward comparison possible. How our experimental design incorporates these features of the alternative plans is discussed below.

Previous experiments have been done studying mechanisms for allocating rights to use the rails (e.g., Brewer, 1997a,b; Brewer and Plott, 1996; Isacsson and Nilsson, 1997; Nilsson, 1997). These studies are discussed in the following section. They are primarily concerned with allocation of rights to use the infrastructure. Because we are mainly concerned with studying the implications of choosing between competition for and competition on the rails, our experiments differ significantly from previous ones. The most notable difference is that we study both the allocation of rights and the scheduling and pricing of trains. This is done at the cost of paying less attention to the question of designing an optimal auction to use for allocating the rights, which was the main focus of previous studies. Thus the research programs are complementary since their focus is on different aspects of the rail network privatization problem.

The rest of the paper proceeds as follows. Section 2 describes properties of the Dutch railway network that should be included in the experimental environment and discusses previous experimental work on similar networks. Section 3 presents the network model used in the experiments. Section 4
describes the experimental route scheduling and pricing tasks and the experimental design and procedures, including the combinatorial auction used to allocate routes in competition on the rails. Section 5 reports analysis of data from the experiments. Section 6 contains concluding remarks.

2. The Rail Network Environment

A. Characteristics of the Dutch Railway Network

We start with an overview of the characteristics of the Dutch railway network that should be incorporated into an experimental design. These characteristics are described by the following seven points.

1. The infrastructure is owned and maintained by the government.
2. Each combination of a route and time slot can be seen as a marketable good. There are substitutabilities and complementarities in demand for these transportation goods.
3. The cost structure includes fixed and variable costs. The fixed costs reflect the leasing (or depreciation) of trains. The variable costs can include fees for use of the infrastructure.
4. Marginal costs for additional passengers are 0, with “spikes” at transportation levels where additional cars have to be added to a train. The marginal cost is infinite where the maximum train length is reached.
5. Passenger ticket prices can differ between peak and off-peak time slots.
6. Operators know the demand structure.
7. The demand for transportation is:
   • negatively related to the price of transportation;
   • negatively related to travel time;
   • higher if there are connecting trains (station complementarity);
   • lower if there are other trains on the same routes in adjacent time slots (time-slot substitutability);
   • higher in some time slots than in others.

B. Previous Experimental Studies

We next consider previous experimental studies of railway networks. In these studies the structure is as follows. A “point of departure” is a set of possible trains that could run on the rails. Some of these trains are in conflict: they cannot simultaneously be allocated. Participants in the experiment represent operators. They are given redemption values for the various trains. If they are allocated the rights to a train, it is automatically assumed that the train will be run and operators earn
the redemption value attributed to that train. The objective of previous studies has generally been to study how various auction mechanisms perform in dealing with route conflicts between trains while efficiently allocating the rights.

The binary conflict ascending price (BICAP) procedure introduced by Brewer and Plott (1996) admits multiple rounds of bidding. The standing bids at any round are the ones that yield maximum revenue from amongst all subsets of bids that contain no conflicting routes. In their experiments, the BICAP mechanism produced highly efficient allocations of rights. Brewer (1997a) added a secondary computerized market to support agents in finding feasible, revenue-increasing route assignments. His initial experiments provide support for the effectiveness of this smart market. Brewer (1997b) discusses extensions of the mechanism applications to network environments with externalities and public goods. These extensions make it possible to incorporate the revenue implications of station complementarity and time-slot substitutability in the redemption values.

In a similar setting, Nilsson (1997) reports experiments with a multi-round Vickrey-type auction of routes. The auction mechanism imposes non-collision feasibility constraints. The pricing rule charges a winning bidder the highest aggregate value of all bids that his combinatorial bid displaces. The reported experiments generated allocations of rights that were 90-100% efficient. Isacsson and Nilsson (1997) extends this experimental research to include four types of auction markets constructed by crossing first-price and second-price auction pricing rules with one-shot (or sealed-bid) and ascending-price (or multiple-round) bidding procedures.

C. Experimental Design Differences

The structure of these previous experiments was well suited for the problems addressed in those studies. That structure also allows one to capture some, but not all, of the characteristics of the Dutch railway network described above. The infrastructure is given. Costs, and market synergies (substitutability and complementarity) can be incorporated in the redemption values, as can peak and off-peak demand. The characteristics related to prices are not captured in this setup, however. In addition, note that the synergies are imposed if one applies them in this way. A different matter is whether operators are capable of actually taking account of these synergies when scheduling trains. For our study, this coordination problem may be very important, because the coordination difficulties may be different in competition on and for the rails.

Therefore, in our experiments we have decided to split the allocation of rights from the scheduling and pricing of trains. We believe that the present paper is the first to report experiments that involve both allocation and scheduling and also the first to experimentally compare competition for
and competition on the rails. In introducing these features of the rail network environment into our experiments, we have simplified other aspects of the network design. First, we have kept the auction used to allocate rights as simple as possible, keeping in mind the tradeoff between simplicity and efficiency. We focused on the comparability of the auctions in for and on the rails (in terms of number of rounds, stopping rules, etc.) and on some basic characteristics that should support efficiency of the auction. Second, to simplify the environment, we assume a common value structure: the demand structure and costs are the same for all operators. Though we believe this to be a reasonable assumption for the present problem, it is not a necessary characteristic of our design. In future research the demand structure can be varied across subjects in a straightforward way.

3. A Simple Stylized Model of a Rail Network

Our experimental network is kept as simple as possible to still be able to capture the central features of a rail network. It distinguishes regions, stations, and routes between stations. A “region” is a set of stations where there is a concentration of demand by travelers. A “station” is a place travelers can embark and disembark. A “route” provides a link between stations.

Time is divided into “time slots.” The network is double tracked; hence it is possible to run at most one train in each direction on any route in any time slot. Travel from one station to another within a region takes one time slot. Travel between adjacent stations in different regions also takes one time slot.

The network includes time substitutability and node complementarity. “Time substitutability” means that travel demand for the route from station $i$ to station $j$ at time $t$ is a gross substitute for travel demand for that route at other times. “Node complementarity” means that travel demand for the route from station $i$ to station $j$ at time $t$ is a gross complement of travel demands for routes that terminate at station $i$ at time $t-1$ and for routes that emanate from station $j$ at time $t+1$.

A. The Network

The network consists of two regions, $A$ and $B$. Each region has three stations, 1, 2 and 3. There is one double tracked route between every adjacent pair of stations. There is one such route between

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1In short, we use a (combinatorial, in case of competition on the rails) auction with a high bid (or first-price) pricing rule and ascending-bid, multiple rounds of bidding. We do not experiment with alternative auctions, as do Isacsson and Nilsson (1997). The possibilities for maintaining comparability between the auctions was restricted by the Ministry’s demands.

4In the experiment, “regions” were called “areas”; “stations” were called “nodes”; “routes” were called “connections”; “trains” were called “carts”; and “passengers” were called “products”. 

5
the two regions \((A_3B_3 \text{ and } B_3A_3)\) in Figure 1).

Besides these routes between adjacent stations, in both regions there is a direct route between stations 1 and 3. This allows an express train to travel on the same rails as a local train would travel from 1 to 2 or 2 to 3. It travels twice as fast however. As a consequence, if there is an express train going from 1 to 3, it is not possible that there is a local train going from 1 to 2 or from 2 to 3 at the same time. Similarly, if there is a local train going from 1 to 2 or a local train going from 2 to 3, it is not possible that there is an express train going from 1 to 3 at the same time. These restrictions hold for both regions and in both directions.

The use of a train in different time slots is constrained through physically logical restrictions. At any station, in any time slot, a new train may be leased and used (yielding fixed costs and variable costs). A train that was used in any previous time slot may be used again (yielding only variable costs) if it is available at the station it is assumed to depart from. A train is available at a station if it was brought there from its original point of departure using only routes that the operator concerned has the rights to. Finally, for simplicity, we will assume that trains have no capacity constraints. This implies that the marginal costs for an additional passenger are zero for any quantity of travel, making it possible to determine the efficient allocation (see below).

Summarizing, the following network with seven two-way routes is used in the experiment (where a line describes a two-way route):

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5 We believe that assuming no capacity constraints will not affect the comparison between competition for and on the rails.
Use of the network is divided into $T$ time slots. Except for the restrictions mentioned above (concerning express trains and local trains), any trains can travel simultaneously on all routes (in both directions) in any time slot. Only one train per direction can travel on any single route in any given time slot, however. The rights to use routes are assigned in the allocation part of an experiment. The allocation procedures take the restrictions into account; thus if a right to an express train is allocated, no other rights involving that route in that time slot are allocated.

**B. Demand Structure and Efficient Allocation**

In the experiment, subjects play the role of providers of passenger services. Demand for these services is simulated in a manner that is known to the subjects. This is done as follows. To determine the actual travel on any route, we start with a baseline travel that varies across routes but is the same in time slots 2…$T$. In time slot 1, the baseline travel is twice the level in other slots. This represents a peak hour.\(^6\) This baseline travel is adjusted in various ways to determine actual travel on the routes: (a) if no train is run on a route, the actual travel is zero; (b) actual travel on a route decreases with an increase in price for that route; (c) actual travel is lower than baseline travel if another train traveled the same route in the previous time slot or if another train will travel that route in the next time slot; (d) actual travel is higher than baseline travel if there is a connecting train arriving at the departure station (on a different route) in the previous time slot; and (e) actual travel is higher than baseline travel if there is a connecting train leaving from the arrival station (on a different route) in the following time slot.

The following equations are used to determine actual travel, $Q_{ij}^t$, on the routes from $i$ to $j$:

\[^6\] Note that the baseline demand is assumed to be equal across subjects. This gives the common value structure referred to above.
(where \(i,j \in \{A_1, A_2, A_3, B_1, B_2, B_3; \ i \neq j\}\)) in time slots \(t=1,2,\ldots,T\), if a train is running:

\[
Q^t_{ij} = \max\{0,V^t_{ij} - p^t_{ij} - \alpha \cdot (Q^{t-1}_{ij} + Q^{t+1}_{ij}) + \beta \cdot (\sum_{k \in C^i_{ij}} Q^{t-1}_{ki} + \sum_{j \in C^j_{ij}} Q^{t+1}_{ij})\},
\]

where \(V^t_{ij}\) is the baseline travel, \(p^t_{ij} (\geq 0)\) denotes the price charged for route \(ij\) in time slot \(t\), \(C^i_{ij}\) denotes the set of all routes in the same direction as \(ij\) that terminate at \(i\) or emanate from \(j\), and \(\alpha (>0)\) and \(\beta (>0)\) are parameters to be chosen. A route is in the same direction as \(ij\) if both this route and \(ij\) are a move in the clockwise (counter clockwise) direction in Figure 1. The term involving \(\alpha\) gives time substitutability: there is less travel if the same route was traveled in an adjacent time slot. The term involving \(\beta\) gives station complementarity: there is more travel if there is a connecting train in the same direction in either the previous or the next period.

Given the parameters, one needs to determine the actual travel for any given set of routes being run and prices being charged. Denote by \(Q\) the vector containing all \(Q^t_{ij}\)'s. Furthermore, let \(d^t_{ij}\) be a dummy variable indicating whether or not a train is being run on route \(ij\) in time slot \(t\) and let \(D\) denote the vector of \(d^t_{ij}\)'s (ordered in the same way as \(Q\)). Finally, let \(P\) denote the vector of prices \((p^t_{ij})\). The set of equations in (1) can then be written as

\[
Q = F(Q(D,P)).
\]

For any given \(D\) and \(P\), one has the fixed-point problem, \(Q = F(Q)\). \(F\) is a continuous function. Hence, there is at least one solution to this set of equations if the domain is convex and compact. The domain is clearly convex, and it is compact because of the restrictions, \(Q \geq 0\) and \(P \geq 0\). Hence, for given \(D\) and \(P\), we can determine \(Q\).

Given the parameter values chosen for the demand side, the efficient allocation can be determined once the supply side parameters have been chosen. There are fixed costs, \(c^1\), related to leasing a train and variable costs, \(c^2\), for every route on which a train runs. Efficient pricing always implies \(P = 0\), because we are assuming that the trains are large enough to cover the resulting demand (the marginal cost of transporting an additional passenger is equal to 0). To find the efficient allocation, we then determine the vector \(D\) that maximizes net surplus (which is consumer surplus minus total costs).
Consumer surplus for the efficient allocation with zero prices is determined as follows for a given $D$. First, the set of equations (2) is solved for $P$. This gives, for any given route, the price (marginal benefits) as a function of the actual travel in that market and of the prices in other markets. We then fix the prices in other markets at zero. For each market this gives the marginal benefits as a function of the quantity of travel. The consumer surplus per market is the total benefit when the actual travel is determined for a zero price.

To determine consumer surplus when prices are positive, one proceeds as follows. For each route and time slot, fix the prices of other routes and slots at the chosen levels. Consumer surplus on a route and slot is then equal to the total benefit for the actual travel minus the fares paid. This is illustrated for the case of one route and two time slots in Figure 2. Actual quantities of travel in the two time slots are denoted by $Q^1$ and $Q^2$. Because they are gross substitutes (and obey the law of demand), an increase in the price on market 2 will decrease $Q^2$ but will also cause the demand curve for $Q^1$ to shift outward. To determine the “location” of the two demand curves, we fix them at the location determined by the price (and quantity) actually chosen in the other market. In this example, total consumer surplus is equal to the area of triangle A plus that of triangle B.

![Figure 2. Consumer Surpluses in Two Markets](image)

Producer surplus is negative and equal to total costs in the efficient allocation with zero prices. With positive prices, producer surplus is equal to the fares charged minus the total costs. The procedure for calculating the efficiencies in the experiments is more fully explained in Appendix 1.
4. Experimental Design and Procedures

Our experiments study the behavior of subjects on the simple network presented in the previous section. It will be argued below that this network captures the most important characteristics of the Dutch railway system. Subjects are split in groups of 4 that remain constant throughout the experiment.\(^7\) The experiments include multiple rounds, each with two parts. In part A of a round, the rights to schedule trains on various routes are allocated. In part B, operators decide what routes to schedule (an operator can only schedule on a route she has the rights to) and, in one treatment, what prices to charge passengers for the scheduled routes. The monetary unit used in the experiments is the experimental franc. It was known to the subjects that these would be converted to Dutch guilders at a rate of 300 francs = 1 guilder. At the time when the experiments were run, 1 guilder = $0.50. In each round, subjects were given a lump sum payment of 3000 francs.

A. Allocation of Rights

The allocation of rights is made by auctioning them in part A of each round. Two methods are used. In experiments with competition on the rails, each route/time-slot combination is a separate good to be obtained in the auction. Given the synergies in demand (actual travel depends on what happens in other route/time slots), it is important to use a combinatorial auction in which bidders can submit “all or nothing” bids on any feasible combination of routes. We use a multi-round combinatorial auction in which bidders can submit bids on any combination of routes and time slots. The bidders can revise their bids while the auction is open. In addition, they can submit multiple bids in any single round of bidding. When the market closes, the auction uses a first price rule for calculating market prices; that is, winning bidders pay the amounts of their bids.

In experiments with competition for the rails, subjects bid on monopoly rights for regions A and B and for the interregional route. They submit price bids for the trains they will schedule. Bids take the form of one peak and one off-peak price that apply to all routes. Bids are ordered by weighting the peak price by 1 and the off-peak price by 2 and summing. These weights reflect the relative magnitudes of baseline travel in the peak and off-peak time slots. The lowest bid for each region that results from this aggregation is the winning bid for those rights. In this case, we did not use a combinatorial auction. Given the fact that we only distinguished three regions and that subjects could submit simultaneous bids, we thought that combinatorial bidding would be an unnecessary complication.\(^8\)

This specific allocation and pricing procedure was requested by the Ministry. Apart from the

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\(^7\)At most 4 groups participated in any single session.
differences in the two mechanisms, we kept the auction rules as similar as possible. In both cases, a maximum of seven bids per round was allowed. In addition, a maximum of ten bidding rounds was held. After round 3, no new bidding round was started if the previous standing bids were not improved. These rules were common information.

B. Route Scheduling and Pricing

In competition for the rails, winning bidders are required to run trains on the routes and time slots in a minimum schedule. They may also run trains on routes and time slots that are not in the minimum schedule but must charge the peak and off-peak fares that they bid in the auction for all trains, whether in the minimum schedule or not. Because the particular minimum schedule imposed by the government might affect the outcome of the experiments, two alternative minimum route schedules are used as experimental treatments. In one treatment, the minimum schedule was relatively efficient (EF) and in the other it was relatively inefficient (IE). The actual minimum schedules used are presented in Appendix 2.

In part B of a round, operators have to determine schedules for running trains on the parts of the network for which they obtained the rights. A schedule consists of a decision whether or not to run a train for every allocated right in every time slot. Scheduling is done with the aid of a computer program that imposes the network feasibility (or train non-collision) constraints. For example, consider the case where in the bidding part of an experiment a subject obtained the rights for routes $A_1A_2$ and $A_2A_3$ in time slot $t$ (note that these routes are always allocated to one subject in competition for the rails). In that case, the scheduling software would permit the subject to schedule local trains on routes $A_1A_2$ and $A_2A_3$, or an express train on route $A_1A_3$, for time slot $t$. The software would not allow scheduling of $A_1A_3$ together with either $A_1A_2$ or $A_2A_3$.

In experiments with competition for the rails, operators must run trains on the routes in the minimum schedule but can add trains on other routes. In experiments with competition on the rails, operators are free to schedule or not schedule trains on any routes for which they obtained the rights. In addition, in the scheduling part of experiments with competition on the rails, operators need to choose peak and off-peak prices for those routes on which they schedule trains.

Operators schedule trains and (where applicable) set prices without knowing the decisions of other operators for other parts of the schedule. Once a schedule has been set, travel is simulated and

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8 In a pilot study where a combinatorial auction was used for this case, virtually no combinatorial bids were made.
9 We did not provide a minimum schedule that is a proper subset of the efficient schedule to be presented below. This way, we do not push the subjects towards efficiency. Moreover, both minimum schedules can be developed in a perfectly symmetric way (see Appendix 2).
this determines the earnings of the operators.

After scheduling and (where applicable) pricing is completed, operators are told their earnings. They are then allowed to make a new schedule for the same allocation of rights. This is used to determine the earnings again. For each allocation of rights, the scheduling is done twice in competition on the rails and six times in competition for the rails.\textsuperscript{10} Then, a new round is started where part A is used to allocate rights and part B is again undertaken several times. The two-part rounds are run three times in every condition.

\textbf{C. Operator Earnings}

Operators make money by transporting (simulated) passengers. The amount they make depends on the number of travelers they transport on various routes and time slots, the prices they charge, and the costs of running a train. These costs have two elements. First, there are fixed costs of leasing trains. Once a train is leased, it can be used for different routes in different time slots, as long as this is physically possible in the way discussed above. Second, there are variable costs of running trains on routes. These variable costs are independent of the number of passengers transported (\textit{i.e.}, the marginal costs for an additional passenger are equal to zero). A subject’s earnings are equal to her revenues (the summation of prices times numbers of travelers on various route and time slots) minus the fixed and variable costs.

\textbf{D. Subjects}

Subjects in the experiment represent the train operators. They are recruited from the undergraduate population of the University of Amsterdam. Subjects are first brought in for a three hour training session. In this session they practice making schedules (including price schedules). These training sessions consist of 3 phases. The first two are only concerned with part B of the experiment. Subjects are paid a fixed fee of 60 guilders ($30) for participation in the training session. This fee is paid after the actual experimental session has taken place.

In phase 1, subjects are allocated the rights to use all routes and time slots. They are allowed to try all kinds of combinations of routes and prices to see what the effects are. The decisions are not recorded.

In phase 2, we preset the trains scheduled on several routes and time slots and let the subjects decide on the remaining ones. Before they make their schedules, we inform them of which routes and time slots they are not responsible for scheduling. Then they have to make a schedule of trains for

\textsuperscript{10}The difference in the number of scheduling rounds was at the explicit request of the Ministry. It reflects
their own routes and time slots knowing what trains have been scheduled on the other routes and time slots in the pre-set schedule. They do so for each of the preset schedules within 45 minutes. In this phase, subjects’ earnings are recorded. These are not paid to the subjects but used to select subjects for participation in the experiment itself.

In phase 3, the subjects practice the auction mechanisms to be used in part A of the experiments. The subjects that make the most money in phase 2 are allowed to participate in the actual experiment. This procedure was common information.\textsuperscript{11}

\textit{E. Network, Demand, and Cost Parameters}

We use five time slots; that is, $T = 5$. As described above, we start with a baseline number of travelers when determining actual travel. This baseline number varies across routes but is the same in time slots 2...5. Time slot 1 is the peak demand slot. In time slot 1, the baseline travel is twice the (common) level in the other slots.

The baseline travel quantities, $V_{ij}^t$, are equal for both track directions connecting any two stations; that is, $V_{ij}^t = V_{ji}^t$, for all $t$, for all $i,j$. Figure 3 shows the baseline travel quantities that we use in the experiments for time slots 2-5. For, example, $V_{12}^t = V_{21}^t = 60$ for $t = 2,...,5$. Baseline travel quantities for time slot 1 are twice the amounts in Figure 3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{baseline_travel_quantities.png}
\caption{Baseline Travel Quantities}
\end{figure}

Recall from our discussion of the structure of demand in equation (1) that the term involving $\alpha$ gives time substitutability: there is less travel if the same route was traveled in an adjacent time slot. The term involving $\beta$ gives station complementarity: there is more travel if there is a connecting train in

\textsuperscript{11} About 75\% of the participants in the training sessions participated in the experiment itself. The others were told at the start of the experiment that they could not participate, paid the fee for the training session, and sent off.
the same direction in either the previous or the next period. We use the specific values, $\alpha = 0.2$ and $\beta = 0.3$ in the experiments.

The specific values of the cost parameters used in the experiments are as follows. The fixed cost of leasing a train are set at $c^f = 2000$ francs. The variable cost of running a train on any route is set at $c^v = 300$ francs.

\section*{F. The Efficient Schedule for the Design Parameters}

The efficient schedule for the design parameters is identified with an optimization algorithm that implements the following procedure. First, all prices are set equal to zero. Then a randomly-generated vector of 70 zeros and ones is entered as the initial realization of the current route/time slot schedule. A zero (respectively, one) entry in the vector indicates that a train is (respectively, is not) run on the indicated route/time slot. The zero prices and the current schedule determine the current actual travel according to equation (1). Then the minimum number of trains (i.e. realizations of fixed cost) required to meet the current schedule is calculated. Current travel, minimized fixed cost, and variable cost of the current schedule are used to calculate the current schedule’s efficiency (sum of consumer and producer surplus) following the logic explained in section 2.B. Then a local change in the vector of 70 zeros and ones is introduced as the updated current schedule and the preceding calculation steps are carried out. If efficiency increases, the updated current schedule is adopted. If efficiency is not increased, the current schedule is not changed. Next, a new local change in the current schedule is introduced and the resulting efficiency is calculated, the change is adopted or not, and so on. Periodic random deviations in the schedule are introduced so as to make it unlikely that the optimization algorithm will get stuck at a local maximum.

![Figure 4. The Efficient Solution](image)

The $V^t_q$, $c^f$, $c^v$, $\alpha$, and $\beta$ parameter values in our experimental design imply the efficient solution portrayed in Figure 4. A short arrow denotes a local train running in the direction of the arrow.
A long arrow denotes an express train. Thus the period 1 efficient schedule has local trains running on the routes $A_1A_2$, $A_2A_3$, and $A_3B_3$ and express trains running on the routes $B_1B_3$ and $B_3B_1$. The efficient schedule uses 5 trains and has a number of interesting features. It does not favor either competition on or competition for the rails; in both treatments this schedule can be reached. In one of the two regions, express trains are run in period 1; in the other region they are not. In periods 2 and 4, the local trains run in opposite directions than they do in periods 3 and 5.

The two minimum schedules used in competition for the rails were derived from this efficient schedule. They are presented in Appendix 2. In both cases, three trains are used. In the relatively efficient schedule (EF) there is an express train going back and forth on $B_1B_3$ and $B_3B_1$, a train connecting to this express train and going back and forth on $A_4B_3$ and $B_3A_4$, and a local train moving through region A (connecting to the train on $A_4B_3$ and $B_3A_4$, when it arrives at and leaves from $A_4$). In the relatively inefficient schedule (IE), there is an express train going back and forth on $A_4A_3$ and $A_3A_4$, a train not connecting to this express train and going back and forth on $A_4B_3$ and $B_3A_4$, and a local train moving through region B (not connecting to the train on $A_4B_3$ and $B_3A_4$ when it arrives at and leaves from $B_3$). If no other trains are scheduled and prices are set at zero, the efficiencies of these minimum schedules are 61.8% for EF and 15.2% for IE.

G. Relating the Experimental Design to the Main Characteristics

In section 2, we listed the main characteristics of the Dutch railway network. Now we can compare these with the characteristics of the experimental railway network. First, the infrastructure is given for the subjects. Next, the setup treats each route/time slot as a marketable good and allows for complementarities and substitutabilities. In addition, the cost structure distinguished fixed and variable costs and sets the marginal costs for an additional passenger equal to zero. Prices may be different in peak period 1 than in off peak periods 2-5. The demand structure is known to operators and equation (1) captures the elements of the demand structure distinguished in Section 2.
H. Summary of the Sessions

Figure 5 summarizes the experimental treatments.

ON: 2 sessions; 7 groups (3 auctions, 2 scheduling rounds each)

EF: 2 sessions; 7 groups (3 auctions, 6 scheduling rounds each)

FOR: 4 sessions

IE: 2 sessions; 7 groups (3 auctions, 6 scheduling rounds each)

Figure 5: Summary of the Sessions

ON = competition on the rails; FOR = competition for the rails; EF = relatively efficient minimum schedule; IE = relatively inefficient minimum schedule.

5. Experimental Results

We compare the performance of competition on the rails with both the relatively efficient and relatively inefficient treatments of competition for the rails. Performance evaluations are based on several criteria: the lowest weighted price charged to passengers; the number of scheduled trains and the number of passenger trips; and the highest efficiency level as well as the distribution of realized surplus.

Recall that in competition for the rails the rights to schedule are auctioned 3 times. For each allocation, subjects provide 6 schedules. Thus, in total 18 schedules per group are produced in competition for the rails. In contrast, in competition on the rails subjects provide only 2 schedules for each of the 3 times the rights are allocated. In total, we have 6 schedules per group in the treatment concerning competition on the rails. In the analysis, we sometimes refer to one schedule as one “year.” Thus, we have observed 18 years in competition for the rails and 6 years in competition on the rails. This difference reflects the Ministry’s plans with respect to the number of years for which licenses would be granted in the alternative privatization policies.

A. Peak and Off-Peak Prices

One item of central concern in choosing between privatization policies is the price of travel. Table 1
reports the prices observed in our experiments for competition on the rails (ON) and competition for the rails with the relatively efficient minimum schedule (FOR\_EF) and the relatively inefficient minimum schedule (FOR\_IE). The figures reported are quantity-weighted prices, averaged across groups and years. Competition on the rails produced higher prices for both peak and off-peak travel than did either treatment with competition for the rails. If we use average weighted price per group as the unit of observation, there are significant differences between both ON and FOR\_IE (Mann-Whitney m=7, n=7; p=0.00) and ON and FOR\_EF (Mann-Whitney m=7, n=7; p=0.00).\textsuperscript{12} Competition for the rails with the inefficient minimum schedule produced higher prices than it did with the efficient schedule, but the difference between the two for-the-rails treatments does not reach conventional significance levels and is much smaller than the difference of either of them from the on-the-rails treatment.\textsuperscript{13}

Table 1. Weighted Prices, Averaged Across Groups and Years*

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Period 1 (peak)</th>
<th>Periods 2-5 (off-peak)</th>
<th>All Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>53.75</td>
<td>24.03</td>
<td>33.48</td>
</tr>
<tr>
<td>FOR_EF</td>
<td>20.59</td>
<td>12.72</td>
<td>15.02</td>
</tr>
<tr>
<td>FOR_IE</td>
<td>24.15</td>
<td>13.77</td>
<td>17.06</td>
</tr>
</tbody>
</table>

*Note: prices are denoted in francs; weighted average price = $\frac{\sum_{j} P_j Q_j}{\sum_{j} Q_j}$

Table 1 reports prices that are averaged over both groups and years within treatments. In order to fully understand the price differences between treatments, it is necessary to separate these two types of averaging. Do average prices in the on-the-rails experiments start high but then decrease, so that the difference from the for-the-rails average prices disappears with subject experience, or are the price differences stable over time? Figure 5 presents graphs of the time series of average prices. We observe that the average price differences among treatments are stable over time.\textsuperscript{14}

\textsuperscript{12} All reported tests use averages per group as data-points. Unless indicated otherwise, the same results are obtained if attention is limited to the first 6 years in the for-the-rails experiments.

\textsuperscript{13} The difference between FOR\_IE and FOR\_EF is significant if we only consider the first six years.

\textsuperscript{14} The fact that prices are almost constant within each round of 6 years in the for-the-rails experiments is to a large extent explained by the fact that subjects have to charge the same price for each year of a round. Nevertheless, small shifts in weighted average price may result if the number of transported passengers change within a round.
B. Train Service and Passenger Trips

The frequency with which trains run on the network (the number of route/time slots scheduled) and the number of passengers transported are important considerations in choosing a privatization policy. Table 2 reports the average number of route/time slots in which trains were scheduled, and the average number of passengers that traveled on these trains, for each of the three treatments. Significantly more route/time slots were scheduled in both the on-the-rails experiments and the for-the-rails experiments with the inefficient minimum schedule than in the for-the-rails experiments with the efficient minimum schedule (Mann-Whitney ON versus FOR$_{EF}$: $m=7$, $n=7$; $p=0.01$; Mann-Whitney FOR$_{EF}$ versus FOR$_{IE}$: $m=7$, $n=7$; $p=0.01$). Nevertheless, despite the small number of routes scheduled in for-the-rails experiments with the efficient minimum schedule, the total number of transported passengers per schedule is higher than in both the on-the-rails experiments and the for-the-rails experiments with the efficient minimum schedule. In the for-the-rails experiments with the efficient minimum schedule there is a higher number of transported passengers per train ride than in the for-the-rails experiments with the inefficient minimum schedule. In the on-the-rails experiments trains are relatively most empty.

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15 The difference between FOR$_{EF}$ and FOR$_{IE}$ is significant at the 10% level if only the first 6 years are considered.
Table 2. Actual Average Quantities and Routes per Schedule

<table>
<thead>
<tr>
<th>Treatment</th>
<th>No. of Route/Time Slots Scheduled</th>
<th>No. of Transported Passengers</th>
<th>Ave. No. of Passengers per Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>27.07</td>
<td>1073.12</td>
<td>39.64</td>
</tr>
<tr>
<td>FOR_EF</td>
<td>21.87</td>
<td>1518.55</td>
<td>69.44</td>
</tr>
<tr>
<td>FOR_IE</td>
<td>27.57</td>
<td>1274.90</td>
<td>46.24</td>
</tr>
</tbody>
</table>

C. Allocative Efficiencies

Economic efficiency is an important criterion for choosing between privatization policies. Table 3 reports allocative efficiency measures for our experiments. From the second column, we observe that the total realized surplus was largest in the for-the-rails experiments with the efficient minimum schedule and that there was little difference between the surpluses realized in the other two treatments. The third column reports the ratio of the realized total surplus to the surplus implied by the optimal allocation (times 100). Note that the most efficient treatment (FOR\_EF) obtained 64% of the maximum possible surplus.16 The other two treatments obtained 36-38% of the maximum surplus. The surplus in FOR\_EF is significantly higher than in both FOR\_IE (Mann-Whitney: m=7, n=7; p=0.00) and ON (Mann-Whitney: m=7, n=7; p=0.00). The difference in surplus between FOR\_IE and ON is not significant.

Table 3. Observed Surpluses

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Surplus</th>
<th>Actual/Max.Efficiency</th>
<th>Consumer Surplus</th>
<th>Producer Surplus</th>
<th>Government Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>37331</td>
<td>38.2%</td>
<td>21976 (59%)</td>
<td>6453 (17%)</td>
<td>8904 (24%)</td>
</tr>
<tr>
<td>FOR_EF</td>
<td>62459</td>
<td>64.0%</td>
<td>54971 (88%)</td>
<td>7487 (12%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>FOR_IE</td>
<td>34819</td>
<td>35.7%</td>
<td>32991 (95%)</td>
<td>1827 (5%)</td>
<td>0 (0%)</td>
</tr>
</tbody>
</table>

Columns 4-6 in Table 3 report the decomposition of the realized surpluses into those accruing to consumers, producers, and the government. Experiments with competition on the rails generated surplus for the government from the revenue collected from auctioning the rights to use the rails. The two for-the-rails treatments do not generate surplus for the government because bidding in those

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16Note that the maximum aggregate surplus in the efficient schedule (97,622) is not feasible, because at prices equal to zero producers would make losses.
experiments is in terms of prices to be charged to passengers. Note that consumers obtained by far the highest percentage of the surplus in both of the for-the-rails treatments. In the on-the-rails treatment, the government captured more of the surplus than the producers but the highest share was still obtained by consumers.

The surplus measures reported in Table 3 are averaged across groups and over time. As with price measures, one needs to separate these two types of averaging in order to fully evaluate the results. Figure 6 reports the time series of total realized surplus in the three types of experiments. Observe that the allocative efficiency differences among the three treatments were relatively stable over time. Only the surplus realized in ON shows a slightly increasing trend.

![Figure 6: Time Series of Surplus](image)

D. Sources of Inefficiencies

Less than optimal efficiency can be caused by both non-optimal routing of trains and non-optimal pricing of the transportation services of the trains. Table 4 reports the total efficiency losses for the three treatments and their decomposition into the losses attributable to non-optimal routing and pricing. The second column reports the total efficiency losses for the three treatments; these numbers are equal to 1 minus the numbers in the third column of Table 3. The third and fourth columns show the decomposition.
Table 4. Sources of Efficiency Losses

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Efficiency Loss (%)</th>
<th>Due to Routing (%)</th>
<th>Due to Pricing (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>61.8</td>
<td>34.6</td>
<td>27.2</td>
</tr>
<tr>
<td>FOR$_{EF}$</td>
<td>36.0</td>
<td>21.2</td>
<td>14.8</td>
</tr>
<tr>
<td>FOR$_{IE}$</td>
<td>64.3</td>
<td>55.0</td>
<td>9.3</td>
</tr>
</tbody>
</table>

The decomposition is obtained as follows. For each schedule we compute what the realized surplus would have been if providers would have charged a price of 0 francs to consumers for the routes that they actually scheduled. The difference between the maximum possible surplus and this hypothetical surplus is the efficiency loss due to routing. The difference between this hypothetical surplus and the actual surplus is the efficiency loss due to pricing.

Competition on the rails (ON) produced much larger losses from both non-optimal routing and pricing than did competition for the rails with the efficient minimum schedule (FOR$_{EF}$). In fact, the surplus lost in ON due to routing alone is almost as large as the total loss in FOR$_{EF}$. Apparently, subjects have a hard time in coordinating their trains towards an efficient schedule in FOR$_{EF}$. In contrast, ON produced smaller losses from non-optimal routing, and larger losses from non-optimal pricing, than did competition for the rails with the inefficient minimum schedule (FOR$_{IE}$). Also note that about 59% (21.2/36.0) of the efficiency loss in FOR$_{EF}$ is attributable to non-optimal routing. This is close to the 56% (34.6/61.8) figure for ON but very different from the 86% (55.0/64.3) attribution to non-optimal routing for FOR$_{IE}$.

E. Implications of Network Monopoly

As previously noted, in implementing the next stage in privatizing its passenger rail service the Netherlands will be replacing a private unregulated monopolist. Of course, monopoly can result from any of the policy alternatives considered in our experiments but the implications of monopoly can be very different in competition for the rails than in competition on the rails. Table 5 presents a decomposition of price and efficiency loss data according to whether or not the bidding stage of an experiment assigned the rights to the interregional route and both regions all to the same bidder.

In the case of ON, the greater inefficiency due to non-optimal pricing with monopoly (32.7% vs. 25.0%) was more than offset by lower inefficiency due to routing (24.4% vs. 38.6%), with the
result that total efficiency loss was lower with monopoly allocation of the rights. The ON treatment’s lower efficiency loss from non-optimal routing with monopoly assignment of rights can be explained by the monopolist’s greater success in dealing with route/time slot complementarities and substitutabilities in demand.

In contrast, total efficiency losses were larger with monopoly for both \( \text{FOR}_{EF} \) and \( \text{FOR}_{IE} \), and this resulted from the larger inefficiencies from non-optimal routing that occurred with monopoly. This may seem puzzling but can be better understood by keeping in mind that in the for-the-rails treatments without network monopoly there were, by design, regional monopolies. It appears that subjects were better able to handle demand complementarities and substitutabilities when routing trains through one region than when attempting to deal with the complexities of the whole network.

Table 5. Effects of Monopoly in the Network

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Monopoly</th>
<th>Wt. Ave. Price</th>
<th>Efficiency Loss (%)</th>
<th>Due to Routing (%)</th>
<th>Due to Pricing (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>No</td>
<td>33.64</td>
<td>63.6</td>
<td>38.6</td>
<td>25.0</td>
</tr>
<tr>
<td>ON</td>
<td>Yes</td>
<td>33.07</td>
<td>57.1</td>
<td>24.4</td>
<td>32.7</td>
</tr>
<tr>
<td>( \text{FOR}_{EF} )</td>
<td>No</td>
<td>16.45</td>
<td>35.1</td>
<td>18.5</td>
<td>16.6</td>
</tr>
<tr>
<td>( \text{FOR}_{EF} )</td>
<td>Yes</td>
<td>11.44</td>
<td>38.3</td>
<td>27.8</td>
<td>10.5</td>
</tr>
<tr>
<td>( \text{FOR}_{IE} )</td>
<td>No</td>
<td>17.26</td>
<td>63.4</td>
<td>54.2</td>
<td>9.2</td>
</tr>
<tr>
<td>( \text{FOR}_{IE} )</td>
<td>Yes</td>
<td>15.86</td>
<td>70.2</td>
<td>59.9</td>
<td>10.3</td>
</tr>
</tbody>
</table>

6. Summary and Conclusions

There may be several, not necessarily mutually consistent, objectives of privatizing passenger rail service. We have reported measures of results relevant to some such objectives from experiments with a simple stylized rail network. Two alternative approaches to privatization were studied in our experiments: competition on the rails and competition for the rails. Our implementation of competition on the rails consisted of experiments in which subjects first bid in a (first-price sealed-bid) combinatorial auction for rights to schedule trains on the network and then scheduled trains and priced transportation services for the route and time slot rights obtained in the auction.

Experiments with competition for the rails also consisted of two parts. In the first part, subjects bid for monopoly rights in two regions and one inter-regional route. Bids took the form of
prices for supplying a pre-specified minimum schedule; they included a peak-demand price and an off-peak price. Weighted averages of the two prices were constructed and the lowest such average-price bid was the winning bid. Two alternative minimum schedules were included as experimental treatments, a relatively efficient one and a relatively inefficient one. Providers were required to use the prices they bid in the auction not only to run the minimum schedule but also to run other routes. In the second part of a competition for the rails experiment, subjects scheduled trains on the network. They were required to schedule trains on route-and-time slots in the relevant minimum schedule and could schedule additional trains in the region(s) for which they won the monopoly rights in the bidding part of the experiment.

Subjects were paid salient monetary rewards that equaled the difference between revenue from simulated passenger demand and their costs. Revenue was equal to prices times quantities determined by the demand relations reported in equation (1) and explained more fully in Appendix 1. In a competition for the rails experiment, a subject’s costs included fixed and variable costs of scheduled trains. In a competition on the rails experiment, a subject’s costs included accepted bids for route and time slot rights in addition to the fixed and variable costs of scheduling trains.

Which is the best privatization plan for the simple stylized rail network depends upon the assumed objectives of the policy and their relative importance. It also can depend upon on whether or not one assumes that the government or its delegate could implement a relatively efficient, rather than a relatively inefficient, minimum schedule in competition for the rails. As a first step in choosing between privatization plans, it can be informative to rank the plans sequentially in terms of single assumed objectives, as follows. If collecting government revenue were the objective then the preferred plan would (rather obviously) be competition on the rails because the alternative does not yield such revenue. If minimizing rail transportation prices were the objective then the preferred plan would be competition for the rails. If providing many train departures were the objective then the preferred plan would be competition on the rails, unless one assumes that a relatively inefficient minimum schedule would be implemented in competition for the rails. If either economic efficiency or simply transportation of a large number of passengers were the objective, then the preferred plan would be competition for the rails, unless one assumes that the minimum schedule would be relatively inefficient, in which case there is not much difference between the plans.

The research results obtained in our experiments were summarized in the Ministry’s official “plans.” These plans were discussed by government in the first week of March 1998 and immediately sent to Parliament. The government proposal is to choose competition for the rails rather than on the rails. There is no commitment yet with respect to the exact auction rules to be used in the allocation
phase or the minimum schedule to be implemented.
References


Appendix 1. Calculation of Efficiencies in the Experiments

The efficiency numbers that we want to report are

\[ z^k = \frac{E^k}{E^\alpha} \times 100, \]

where \( E^k \) is the realized sum of consumer and producer surpluses in all periods of experiment \( k \) and \( E^\alpha \) is the surplus from optimal pricing and allocation of trains in all time periods.

Producer surplus in a period of experiment \( k \) is simply the sum of all sellers’ revenues minus the sum of all sellers’ costs in that period.

Consumer surplus in a period of experiment \( k \) is calculated as follows. First recall the “actual travel” equations from the paper:

\[ Q^j_i = \max \{0, V^j_i - p^j_i - \alpha \cdot (Q^j_i - Q^r_i) + \beta \cdot (\sum_{k \in C^j_i} Q^j_k + \sum_{j \in C^a_i} Q^a_j)\} \]

There are 70 of these equations, one for each route and time slot. Consumer surplus for a period in an experiment is calculated using the observed quantities \((Q^j_i)\), the observed prices \((p^j_i)\), the experimental parameters \((V^j_i, \alpha, \beta)\), and these 70 equations.

For present purposes, it will help to rewrite equations (2) as follows:

\[Q^j_i = V^j_i - p^j_i - \alpha \cdot (Q^j_i - Q^r_i) + \beta \cdot (\sum_{k \in C^j_i} Q^j_k + \sum_{j \in C^a_i} Q^a_j)\]

\begin{align*}
(3.1) & \quad Q^j_{12} = V^j_{12} - p^j_{12} - \alpha \cdot (Q^j_{12} - Q^r_{12}) + \beta \cdot Q^r_{23} \\
(3.2) & \quad Q^j_{13} = V^j_{13} - p^j_{13} - \alpha \cdot (Q^j_{13} - Q^r_{13}) + \beta \cdot Q^r_{34} \\
(3.3) & \quad Q^j_{23} = V^j_{23} - p^j_{23} - \alpha \cdot (Q^j_{23} - Q^r_{23}) + \beta \cdot (Q^r_{12} + Q^r_{34}) \\
(3.4) & \quad Q^j_{34} = V^j_{34} - p^j_{34} - \alpha \cdot (Q^j_{34} - Q^r_{34}) + \beta \cdot (Q^r_{13} + Q^r_{23} + Q^r_{45} + Q^r_{46}) \\
(3.5) & \quad Q^j_{45} = V^j_{45} - p^j_{45} - \alpha \cdot (Q^j_{45} + Q^r_{45}) + \beta \cdot (Q^r_{34} + Q^r_{56}) \\
(3.6) & \quad Q^j_{46} = V^j_{46} - p^j_{46} - \alpha \cdot (Q^j_{46} + Q^r_{46}) + \beta \cdot Q^r_{56} \\
(3.7) & \quad Q^j_{56} = V^j_{56} - p^j_{56} - \alpha \cdot (Q^j_{56} + Q^r_{56}) + \beta \cdot Q^r_{45} \\
(3.8) & \quad Q^j_{65} = V^j_{65} - p^j_{65} - \alpha \cdot (Q^j_{65} + Q^r_{65}) + \beta \cdot Q^r_{54} \\
(3.9) & \quad Q^j_{64} = V^j_{64} - p^j_{64} - \alpha \cdot (Q^j_{64} + Q^r_{64}) + \beta \cdot Q^r_{43} \\
(3.10) & \quad Q^j_{54} = V^j_{54} - p^j_{54} - \alpha \cdot (Q^j_{54} + Q^r_{54}) + \beta \cdot (Q^r_{65} + Q^r_{43}) \\
(3.11) & \quad Q^j_{43} = V^j_{43} - p^j_{43} - \alpha \cdot (Q^j_{43} + Q^r_{43}) + \beta \cdot (Q^r_{64} + Q^r_{54} + Q^r_{32} + Q^r_{31}) \\
(3.12) & \quad Q^j_{32} = V^j_{32} - p^j_{32} - \alpha \cdot (Q^j_{32} + Q^r_{32}) + \beta \cdot (Q^r_{43} + Q^r_{21}) \\
(3.13) & \quad Q^j_{31} = V^j_{31} - p^j_{31} - \alpha \cdot (Q^j_{31} + Q^r_{31}) + \beta \cdot Q^r_{43} \\
\end{align*}
(3.14) \[ Q'_{21} = V'_{21} - p'_{21} - \alpha \cdot (Q'_{21} + Q'_{21}) + \beta \cdot Q'_{32} \]

Note that is not a problem that both \( Q'_{13} \) and \( Q'_{23} \) appear in equation (3.4), and analogous terms appear in other equations, because of Step 2 below. The following steps calculate the consumer surplus in a period of an experiment.

Step 1: If \( Q'_{ij} = 0 \), then delete the corresponding equation (that has \( Q'_{ij} \) on the left side of the equality) from the set of 70 equations in (3.1) - (3.14) above.

Step 2: In the remaining equations, if \( Q'_{ij} = 0 \) then set it at zero on the right side of all (remaining) equations.

Step 3: Use an algorithm to find a solution of the system of simultaneous equations that result from step 2. These equations should have the \( Q'_{ij} \)’s as dependent variables and the \( p'_{ij} \)’s as independent variables. The solution will be a set of equations with the form,

\[
(4) \quad Q'_{ij} = a'_{ij} + b'_{ij} \cdot p'_{ij} + \sum_{k \neq j, \tau \neq t} c_{k\tau} \cdot p'_{k\tau}
\]

Define

\[
(5) \quad \gamma'_{ij} = a'_{ij} + \sum_{k \neq j, \tau \neq t} c_{k\tau} \cdot p'_{k\tau}
\]

Then equations (4) can be rewritten as

\[
(6) \quad Q'_{ij} = \gamma'_{ij} + b'_{ij} \cdot p'_{ij}
\]

Rewrite (6) as

\[
(7) \quad p'_{ij} = -\frac{\gamma'_{ij}}{b'_{ij}} + \frac{1}{b'_{ij}} Q'_{ij}
\]

Consumer surplus on route \( ij \) in time slot \( t \) in this period is

\[
(8) \quad S'_{ij} = \int_{0}^{Q'_{ij}} (-\frac{\gamma'_{ij}}{b'_{ij}} + \frac{1}{b'_{ij}} x)dx = -\frac{\gamma'_{ij}}{b'_{ij}} Q'_{ij} + \frac{1}{2b'_{ij}} (Q'_{ij})^2
\]

Substitute the observed prices ( \( p'_{k\ell} , k \neq i, \ell \neq j, \tau \neq t \) ) in equation (5) and the observed quantities ( \( Q'_{ij} \) ) in equation (8) to calculate the value of \( S'_{ij} \). Total consumer surplus in this period of this experiment is

\[
(9) \quad CS = \sum_{i,j,t} S'_{ij}
\]
Appendix 2. Relatively Efficient and Inefficient Minimum Schedules

Each of the minimum schedules can be scheduled with 3 trains. The difference in the relative degree of efficiency of the minimum schedules is caused by the fact that the relatively efficient minimum schedule yields some complementarities which are lacking for the relatively inefficient minimum schedule.

Recall the efficient schedule presented in Figure 4.

![Figure A2.1. The Efficient Schedule](image)

The relatively efficient minimum schedule used for the sessions FOR-EF is given by the three trains described in figure A22. With these three trains (and prices equal to zero) an efficiency of 61.8% is obtained. Adding trains can yield substantial efficiency gains. Note that this minimum schedule can be realized even if the regions are allocated to three different operators.

![Figure A2.2. The Relatively Efficient Minimum Schedule (EF)](image)
The relatively inefficient minimum schedule used for the sessions FOR-IE is given by the three trains described in figure A23. With these three trains (and prices equal to zero) an efficiency of 15.2% is obtained. Adding trains can yield substantial efficiency gains. Note that this minimum schedule can be realized even if the regions are allocated to three different operators.

Comparing the EF and IE minimum schedules, note that they are mirror images in the sense that the trains in regions A and B are switched. The substantial effect this has on efficiency is partly due to the differences in the baseline demand in these regions. More important is that the node complementarities in stations $A_3$ and $B_3$ are realized in EF but not in IE.

**Figure A2.3. The Relatively Inefficient Minimum Schedule (IE)**