Attitudes Towards Immigration in the Small Open Economy.

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Abstract :

The purpose of this paper is to explain the attitudes toward immigration and the formation of immigration policy in a small open economy setting including a non-traded sector and imperfect factor mobility across sectors. The direct democracy approach is used to transmit voters' attitudes into immigration policy. I find that the voters regardless of their skill level will be opposed to the inflow of low-skilled immigration and will favor high-skilled immigration, if the domestic non-traded and imported goods are sufficiently poor substitutes in consumption. If the degree of substitution is between non-traded and imported good is high, a country with a high-skilled (low-skilled) median voter will be favorable (opposed) to both low-skilled and high-skilled immigrants. On the economy wide level, higher skill level of voters makes them more tolerant towards immigration.

Key words: immigration policy, non-traded sector, specific factors, imperfect import substitution, general equilibrium, multiple factor ownership, direct democracy, EU enlargement.

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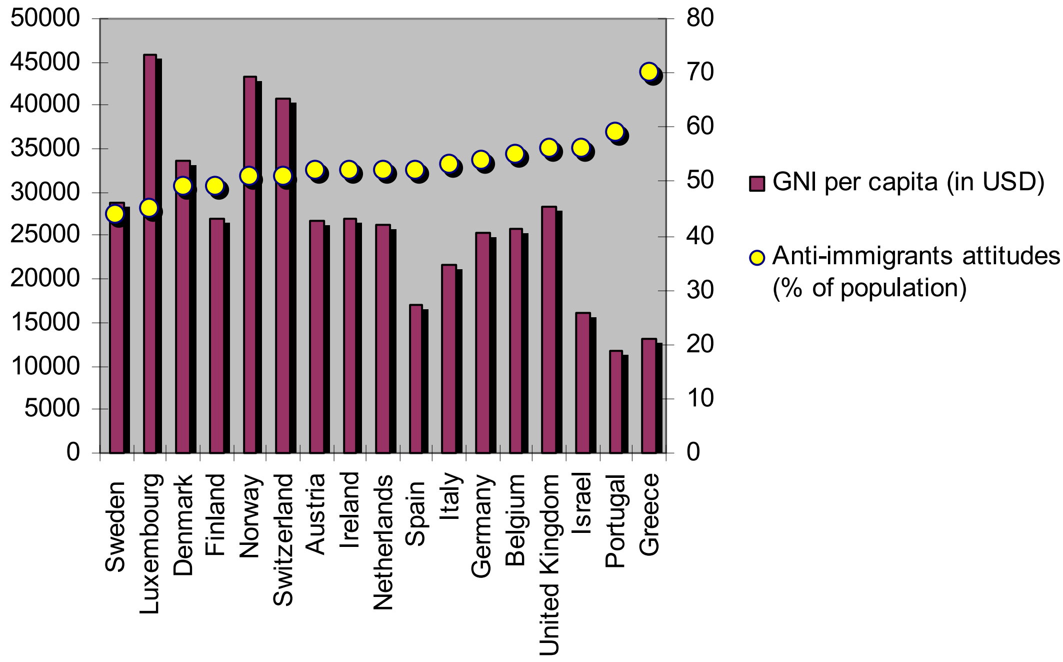
1. Introduction.

For a long time immigration has been one of the most controversial issues on the world economic and political arena. For example, in most present-day European countries, characterized by low internal labor mobility and increasingly ageing population, immigration can be reasonably considered as an important determinant of economic growth and a key element in assuring the sustainability of retirement programs. The new states that have recently joined the European Union constitute an important source of both qualified and low-skilled labor able to fill certain gaps in European economic and social structures. However, several examples of immigration restrictions, the most prominent of which is the imposition of transition periods for free labor movements in the enlarged EU, prove that more complex mechanisms are at work in the determination of actual immigration policies.

As shown in the European Social Survey 2002, on average 54% of European citizens have adopted anti-immigration attitudes, ranging from 44% in Sweden to 70% in Greece. At the same time in many countries we can observe the rise of radical political forces which put the immigration restriction issue at the centre of their program. In a democratic society government's decisions about immigration is the reflection of voters' attitudes toward it. A mix of economic, social and cultural considerations makes a voter feel pro- or anti-immigrant, and none of these factors should be underestimated. Yet mainly economic arguments play a critical role in attitudes formation. This suggests that the principal question to answer is how a particular individual's welfare is affected by foreign labor inflow.

Graph 1 plots the average index of anti-immigration feelings in 2002-2003 (measured on the right vertical axis) and per capita income in 2003 (left vertical axis) for 18 European countries. It suggests that citizens are more tolerant to immigration in high-income countries (Scandinavian states, Switzerland, Luxembourg), and more anti-immigrant in economies with relatively lower per capita income (e.g., Southern Europe). Assuming that higher per capita income corresponds to higher skill level of voters, one should expect a negative correlation between average skill level of individuals and anti-immigration attitudes on the economy-wide level.



Graph 1. Income per capita (2003) and anti-immigrant attitudes (2002-2003)

Source: World Bank, Norris (2005)

However, the attitudes towards all kinds of immigrants are not uniform. Public opinion polls suggest that a negative stance is typically taken on the inflow of low-skilled labor, while the high-skilled foreign individuals do not seem to affect adversely the welfare of domestic residents. At the same time, in most developed countries, and especially in Europe, labor is becoming increasingly immobile and industry specific. Low-skilled workers are more likely to be employed in industries producing non-traded goods, for example, in services and construction sectors, and high-skilled labor is usually specific to export industries.

In this paper I provide some additional insights to the wage-immigration debate, by taking into account the characteristics of many developed economies where a low-skilled non-traded service sector is important, suggesting the appropriateness of the small open economy framework with a non-traded good. I construct a two-sector specific-factor general equilibrium model with a non-traded good. Because adding a non-traded sector complicates considerably the analysis of the model, I make some simplifying assumptions about the model's structure which make it suitable for an economy-wide analysis. First, high-skilled workers are specific to the export industry, while low-skilled labor is employed in domestic non-traded sector. Second, capital is assumed to be mobile across sectors. I identify the elasticity of factor substitution in production and the degree of substitution between domestic and imported goods as key parameters determining the direction of change in nominal factor incomes if immigration occurs. The model's setup allows me to answer how individual preferences are transmitted into concrete policies. Allowing for workers' skill differentiation and unequal "between" group capital distribution, we obtain an immigration policy outcome in a direct democracy framework.

The rest of the paper is organized as follows. Section two presents a brief overview of relevant theoretical literature. Section three develops the production side of the standard specific factor model and lays out the formal derivations of the main equations linking factor rewards, goods prices, and factor endowments. I explain the notion of imperfect substitutability between domestic and foreign imported good, and integrate it into demand side of the model. The analysis of the changes in nominal factor returns and domestic prices follows in section four. In section five I examine utility functions of individuals, and determine political economy outcome in potential receiving country. Section six concludes.

2. Related contributions.

Much of the theoretical literature on attitudes towards immigration is confined to the examination of purely economic arguments as major determinants of attitudes towards foreign labor. Then it is sufficient to explore the evolution of nominal income and consumption prices of a particular individual in order to say whether she will be pro- or anti-immigrant. In the usual price taking small open economy setting, the Heckscher-Ohlin and the Ricardo-Viner models provide important insights and are widely used as starting points in immigration research literature. In the absence of non-traded goods, immigrants can affect households' welfare only through the change in factor income. The Heckscher-Ohlin model, which assumes perfect factor mobility across sectors, thereby taking a long run perspective, predicts that factor returns are insensitive to factor inflows, leaving individuals' welfare unaffected. However, in the medium run Ricardo-Viner model, where along with a mobile factor there are some industry specific factors, immigration of labor lowers the real income of its substitute and raises the real income of all other factors. Relying on information about factor ownership we can predict the attitude of particular individual toward immigration, and consequently, the shape of immigration policy.

Within these two models, several contributions have appended a simple political economy model with attitudes determined by the majority rule in a direct democracy. In all of them, voting is on one issue with single-peaked preferences, and attitude of the median voter depends only on the change of her nominal income, since prices are fixed on world markets[1].

Benhabib (1996) considers an economy producing one good with labor and capital under constant returns to scale. Each individual is endowed with one unit of labor and some amount of capital which is unequally distributed among population. He shows that under direct democracy, any policy which decreases (raises) the overall capital-labor ratio in the economy will be defeated if the median voter's capital ownership is below (above) the critical capital-labor ratio. In other words, if the median individual is capital poor (with respect to a type who is indifferent to immigration) the entry of immigrants, whose relative endowment of capital is lower that the economy's average will be opposed. Similarly, if a median voter is capital-rich she will be unfavorable to the capital-rich migrants. Finally, the amount of capital held by an individual may be interpreted as human capital, allowing us to define voters' attitudes and immigration policy in terms of different skill levels.

A meaningful extension of Benhabib (1996) to an open economy requires that factor incomes respond to immigration. The most natural framework, where factor prices are endogenous, is a Ricardo-Viner model, used by Grether et al. (2001). They assume that two traded goods are produced with two specific factors - high-skilled and low-skilled labor, as well as mobile capital. Each household possesses one unit of labor and a certain amount of capital. Households express their attitude depending on the variation of their income. In the case of infinitesimal immigration (where the aggregate national income does not change with the arrival of foreign labor), the attitude of high-skilled households will be always opposed to

[1] An alternative way to model immigration policy is to follow political contributions approach. See e.g. Facchini and Willmann (2004).

that of low-skilled households. Given that capital is evenly distributed among population, and if sector employing low-skilled workers is labor intensive, domestic low-skilled (high-skilled) workers they are necessarily capital-rich (poor) and the loss from specific factor income is more than (is not) compensated by higher revenue from capital. The outcome will depend on which labor group has the majority. As to sustained immigration, immigration will induce net gains to the economy, making natives more favorable to immigrants. Precisely, the opposition of the low-skilled towards low-skilled immigrants diminishes, and the stance of the high-skilled remains unchanged. At last, authors introduce uneven capital distribution between labor groups (making a realistic assumption that low-skilled households are capital-poor), and show that native low-skilled (high-skilled) individuals become more (less) opposed to low-skilled immigrants, thus rising overall opposition towards immigration[2].

We extend the literature by including a domestically produced non-traded good to the consumption basket of individuals. As its price is endogenously determined, voters' attitude toward immigrants will depend not only on the change of their nominal income, but also on the evolution the purchasing power of their income via changes in of the price of non-traded good. To simplify the analysis, I assume that domestic consumers can buy an imported commodity on the world market, which is an imperfect substitute for domestic non-traded good.

3. MODEL.

In this section I lay down the general equilibrium model, following mainly the works of de Melo and Robinson (1989), and Devarajan et al. (1993). The supply side assumes Ricardo-Viner model characteristics, and is developed along the lines of Jones (1971)[3].

[2] Bilal et al. (2003) obtain similar results in a three-factors two-goods Heckscher-Ohlin model, since factor prices in 3x2 case are endogenous.

[3] Robinson and Thierfelder (2003) develop a similar model, by integrating Heckscher-Ohlin-Samuelson production structure into general equilibrium framework of type de Melo and Robinson (1989). They obtain highly modified magnification effects in the Stolper-Samuelson and Rybczynski Theorems, and show that contrary to the standard HOS model, factor inflows have an effect on factor returns.

3.1. SUPPLY SIDE

Two goods, D and E, are produced in the economy. Good D is consumed domestically, while the entire amount of good E is exported. The subscripts D and E correspond to the sectors where the respective good is produced. Three production factors are available in fixed amounts in the economy: two industry-specific factors, V_D and V_E, and a mobile factor V_N. Let a_{DD} (a_{EE}) be the amount specific factor V_D (V_E) necessary to produce one unit of good D (E). The amount of the mobile factor necessary to produce one unit of the domestic (export) good is equal to a_{ND} (a_{NE}).

Following Jones (1971) and Jones (1975), equations linking the changes in goods and factor prices are[4]:

$$\theta_{DD}\hat{R}_D + \theta_{ND}\hat{R}_N = \hat{p}_D \qquad (1)$$

$$\theta_{EE}\hat{R}_E + \theta_{NE}\hat{R}_N = \hat{p}_E, \qquad (2)$$

where R_N, R_D, R_E are the rewards to the mobile and specific factors, respectively; p_D and p_E are goods' prices; θ_{ij}, $i = D, E, N$, $j = D, E$, is factor's i share in total income generated in sector j, and '^' over a variable denotes the relative change in that variable.

Let σ_D be the elasticity of substitution between factors in industry D, relating the change in the $\frac{a_{ND}}{a_{DD}}$ to the change in the factor price ratio (a comparable definition is applied to the sector E):

$$\sigma_D = \frac{(\hat{a}_{ND} - \hat{a}_{DD})}{(\hat{R}_D - \hat{R}_N)} \quad \left(\sigma_E = \frac{(\hat{a}_{NE} - \hat{a}_{EE})}{(\hat{R}_E - \hat{R}_N)}\right) \qquad (3)$$

[4] The derivation of all equations in this section is provided in appendix II

Formal solutions for the effects on factor returns of changes in commodity prices and factor endowments are provided by the following equations:

$$\hat{R}_D = \left[\beta_D + \frac{1}{\theta_{DD}}\beta_E\right]\hat{p}_D - \frac{\theta_{ND}}{\theta_{DD}}\beta_E \hat{p}_E + \frac{1}{\Delta}\frac{\theta_{ND}}{\theta_{DD}}\left(\hat{V}_N - \lambda_{ND}\hat{V}_D - \lambda_{NE}\hat{V}_E\right) \quad (4)$$

[5]

$$\hat{R}_N = \beta_D \hat{p}_D + \beta_E \hat{p}_E + \frac{1}{\Delta}\left(\lambda_{ND}\hat{V}_D + \lambda_{NE}\hat{V}_E - \hat{V}_N\right) \quad (5)$$

$$\hat{R}_D - \hat{R}_E = \left(\frac{1}{\theta_{EE}}\beta_D + \frac{1}{\theta_{DD}}\beta_E\right)\left(\hat{p}_D - \hat{p}_E\right) + \frac{1}{\Delta}\frac{(\theta_{ND} - \theta_{NE})}{\theta_{DD}\theta_{EE}}\left(\hat{V}_N - \lambda_{ND}\hat{V}_D - \lambda_{NE}\hat{V}_E\right), \quad (6)$$

where λ_{Nj} is the fraction of the mobile factor absorbed by the j-th industry, $j = D, E$;

$$.\Delta = \sum_{i=D,E} \lambda_{Ni}\frac{\sigma_i}{\theta_{ii}}; \quad \beta_{i,i=D,E} = \frac{\lambda_{Ni}\frac{\sigma_i}{\theta_{ii}}}{\sum_i \lambda_{Ni}\frac{\sigma_i}{\theta_{ii}}}.$$

Consider next expression linking the change in production volumes to changes in prices and factor endowments:

$$(\hat{D} - \hat{E}) = \Omega\left(\hat{p}_D - \hat{p}_E\right) + \left(\hat{V}_D - \hat{V}_E\right) + \frac{1}{\Delta}\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)\left(\hat{V}_N - \lambda_{ND}\hat{V}_D - \lambda_{NE}\hat{V}_E\right) \quad (7)$$

where $\Omega = \theta_{ND}\frac{\sigma_D}{\theta_{DD}}\beta_E + \theta_{NE}\frac{\sigma_E}{\theta_{EE}}\beta_D$.

Ω is constant and could be called the constant elasticity of transformation, as in e.g. de Melo and Robinson (1989). It shows the elasticity of substitution along the production transformation schedule. It is positive and finite, and the transformation schedule exhibits the bowed-out shape (is strictly concave to the origin). The larger is its value, the more approaches the transformation schedule a straight line and quicker producers change an

[5] The solution for $\hat{R}_E$ can be obtained by permuting subscripts in the solution for $\hat{R}_D$

output mix in response to change in relative price. In the limiting case the production structure converges to that of Ricardian model, where the economy has a corner solution, producing only one good.

3.2. DEMAND SIDE.

To help keep the model treatable, the so-called "Armington" assumption is used. This assumption whereby consumers differentiate goods depending on their origin is reasonable at the aggregate level and allows one to capture how the representative consumer alters his consumption expenditures when relative prices.

We postulate a simple CES utility function to model the demand for a home and imports good:

$$Q(M,D,\sigma) = \left[\chi M^{(\sigma-1)/\sigma} + (1-\chi)D^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \qquad (8)$$

where Q(.) is utility over the home and foreign good, χ is a parameter that weights the import good relative to the home good, and σ is the constant elasticity of substitution between domestic and imported good[6]. In this model, σ is also the Hicksian price elasticity of demand for imports. The greater is the degree of product differentiation, the smaller is the elasticity of substitution between the products.

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[6] In general, the elasticity of substitution depends on the degree of product differentiation – consumers see goods as imperfect substitutes when there are obvious physical product differences. The greater are these differences, the lower is the elasticity of substitution between the products. However, product differentiation does not depend on actual physical differences alone. Physical identical goods may be differentiated by availability in time, convenience of purchase, after-sales service bundled with the good, of even consumers' perception of inherent unobservable quality. In fact, consumers' perceptions of inherent quality have been shown to be rather strong in some instances. Considering imports, their purchase brings additional transaction costs and risks due to customs paperwork, exchange rate risk, the possibility of disruption of supplies, delayed shipments because of extensive transportation, possibly fewer resources for after sales service on the part of a foreign supplier. A home bias is created, because the domestic industry has advantages in these areas. So, factors not related to physical characteristics may play a particularly strong role in product differentiation, and reflected in the elasticity of substitution. See also Blonigen and Wilson (1999).

Maximisation of (8) subject to the consumers' budget constraint yields:

$$\frac{M}{D} = k\left(\frac{p_D}{p_M}\right)^{\sigma} \qquad (9)$$

where $k = \left(\frac{\chi}{1-\chi}\right)^{\sigma}$ is a constant and p_D and p_M are prices paid by domestic consumers for domestic and import good.

3.3. CLOSING THE MODEL – EXTERNAL AND DOMESTIC SECTORS.

As we treat a general equilibrium model, we require an external closure rule – relationship linking exports and imports together. Let π_m and π_e exogenously given import and export prices, respectively. To simplify the analysis, I set the trade balance to zero, implying that export revenues are entirely spent on import purchases:

$$\pi_M M = \pi_E E \qquad (10),$$

Consider next the domestic price of imports and exports. Choosing the nominal exchange rate as a numeraire, and given that neither tariff nor subsidies are applied, the imports price faced by domestic consumers is equal to world import price $p_M = \pi_M$, and domestic export price is equal to world export price $p_E = \pi_E$.

To close the model, we need to assume that the supply of domestic good is equal to the demand for it, meaning that domestic producers and consumers face the same prices for the domestic good.

Unlike models with pure tradable goods, our model recognizes that the incentive to consume imports versus domestic goods is different from the incentive to produce exports versus domestic good. Therefore, we are confronted with two real exchange rates in the model. Letting e be the nominal exchange rate, the first is the import or demand real exchange rate, $e_m^r = e * \pi_M / P_D = P_M / P_D$, which captures the incentives to consume tradables over nontradables. The second is the export or supply real exchange rate, $e_s^r = e * \pi_E / P_D = P_E / P_D$, which captures the relative profitability of producing for the domestic or export markets.

3.4. INITIAL EQUILIBRIUM.

The endogenous variables to be determined are the domestic price, the amounts of domestic and imported consumption goods, the output mix, and factor returns as a function of exogenous variables (policy instruments) – different levels of foreign production factor inflow. Following de Melo and Robinson (1989), it is convenient to depict the initial equilibrium in a four quadrant diagram.

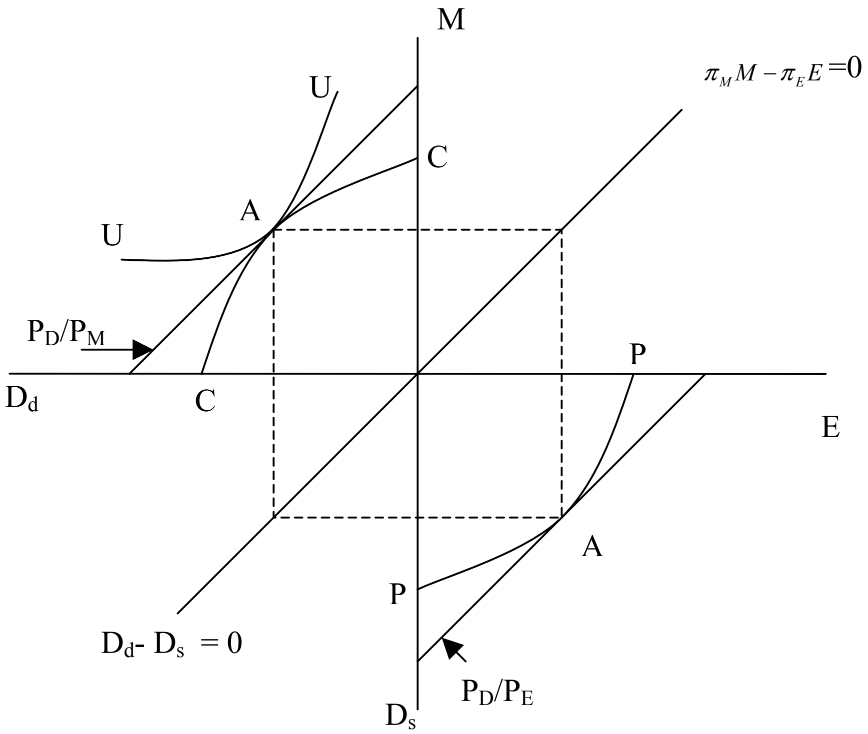


Fig. 1. Initial equilibrium.

In the SE quadrant we depict the production possibility frontier PP. Assuming that initially the trade balance is zero and that exogenous world prices are equal to unity, the foreign offer curve is a 45-degree line in the NE quadrant. A 45-degree line in the SW quadrant defines equilibrium in the domestic good market. A concave to the origin curve CC in the NW quadrant is the consumption possibility frontier is a locus of points that satisfy simultaneously production possibility frontier and balance of trade constraint. Under the assumption of zero trade balance, the CC curve is a mirror image of the PP curve.

Import aggregation function defined by equation 18 is shown through "iso-good" indifference curves UU. The equilibrium (point like A) is achieved at the point where the indifference curve is tangent to the consumption possibility frontier and the consumption price ratio. One can show that at the optimum the marginal rate of substitution in consumption is equal to the marginal rate of transformation in production, i.e., $p_D / p_M = p_D / p_E$ [7].

4. IMMIGRATION AND FACTPOR PRICES.

Before turning to the political economy of immigration, one must establish the relationship between changes factor inflows and changes in factor rewards. I assume that both specific factors are two different types of labor: low-skilled labor, which is used to produce domestic non-traded good, and high-skilled labor, employed in export industry. Capital is assumed to be mobile between industries and is not subject to international movements. In order to concentrate exclusively on labor movements, we set the change in all exogenous variables but labor endowments equal to zero[8].

As we can notice from equation (4) and (5), factor inflows affect directly factor rewards. However, since producer prices also appear as determinants of factor rewards, we should establish the relationship between producer prices and available production factor volumes.

[7] See De Melo and Robinson (1989), p. 64
[8] This implies also that immigrants do not bear any (mobile) capital with them.

The producer price for exportable is exogenous, so we seek to express only domestic price as a function of labor endowments. For this we reconsider equation (7), as well as log differentials of equations (9) and (10), assuming that only labor endowments may change.

$$\begin{cases} (\hat{D}-\hat{E}) = \Omega \hat{p}_D + \left(\hat{V}_D - \hat{V}_E\right) + \dfrac{1}{\Delta}\left(\dfrac{\theta_{ND}\sigma_D}{\theta_{DD}} - \dfrac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)\left(-\lambda_{ND}\hat{V}_D - \lambda_{NE}\hat{V}_E\right) \\ \hat{M} - \hat{D} = \sigma\,\hat{p}_D \\ \hat{M} - \hat{E} = 0 \end{cases} \qquad (11)$$

Solution to the system (11) provides an expression for the domestic price as a function of factor endowments.

$$\hat{p}_D = -\frac{1}{(\sigma+\Omega)}\left(\alpha_D \hat{V}_D + \alpha_E \hat{V}_E\right), \qquad (12)$$

where $\alpha_D = 1 - \lambda_{ND}\dfrac{1}{\Delta}\left(\dfrac{\theta_{ND}\sigma_D}{\theta_{DD}} - \dfrac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)$, $\alpha_E = -1 - \lambda_{NE}\dfrac{1}{\Delta}\left(\dfrac{\theta_{ND}\sigma_D}{\theta_{DD}} - \dfrac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)$. We can prove[9] that the coefficient α_D is always positive, and α_E is negative.

The impact of factor endowments change on the domestic price is straightforward from equation (12). The increase in D specific factor exerts a downward pressure on the domestic price, while the increase in E specific factor pulls it upwards. As the elasticity of import substitution tends to infinity, the effect exerted by labor inflows on domestic price vanishes.

The intuition behind the change in domestic price is the following. Higher amount of low-skilled labor leads to higher output in the domestic sector, and expands both the production possibility and consumption possibility frontiers toward good D. If domestic and imported goods are perfect substitutes, the domestic price stays constant, and it is efficient to produce more of domestic good and less of export good. However, if the elasticity of import substitution takes on some finite value, consumers with higher aggregate income are willing

[9] See appendix II the derivation of all equations of this section.

to consume more of the imported good. For this the economy must produce more exports, and consequently the relative price of domestic to export commodity must fall. The lower is degree of substitution between domestic and import goods, the larger amount of exports has to be produced in order to be exchanged for necessary imports, and consequently, the lower must be domestic price. On the other hand, a higher endowment of high-skilled labor leads to the E-biased expansion of production and consumption possibility frontiers. In the case of perfect import substitution relative prices stay constant, and producers' optimal choice is higher output of good E and lower of good D. If imports and domestic good are not perfect substitutes, individuals demand more of domestic good. The producers respond to it by increasing (decreasing) the output of good D (E), which is possible only if the relative domestic price goes up.

If the two consumption goods are sufficiently poor substitutes (elasticity of substitution is lower than some critical value), higher consumption possibilities due to inflow of either type of labor lead to higher output in both sectors. This is never the case in a standard Ricardo-Viner model, which is a special case of our model with $\sigma \to \infty$: if domestic and imported goods are perfect substitutes, the inflow of D specific factor will necessarily hurt E sector in terms of output and vice versa. A graphical representation of the new equilibrium after the inflow of low-skilled labor for "sufficiently" high and low levels of elasticity of import substitution is provided in appendix I. The case of high-skilled labor immigration is symmetric.

As to the factor rewards, the inspection of equations (4) and (5) shows us that factor endowments intervene twice in solutions for $\hat{R}$'s – once directly, and secondly, through domestic producer price. Substitution of (12) into (4) and (5) yields:

$$
\begin{aligned}
\hat{R}_D &= -\frac{1}{(\sigma+\Omega)}\left[\beta_D + \frac{1}{\theta_{DD}}\beta_E\right]\left(\alpha_D \hat{V}_D + \alpha_E \hat{V}_E\right) + \frac{1}{\Delta}\frac{\theta_{ND}}{\theta_{DD}}\left(-\lambda_{ND}\hat{V}_D - \lambda_{NE}\hat{V}_E\right) \\
\hat{R}_E &= \frac{1}{(\sigma+\Omega)}\frac{\theta_{NE}}{\theta_{EE}}\beta_D\left(\alpha_D \hat{V}_D + \alpha_E \hat{V}_E\right) + \frac{1}{\Delta}\frac{\theta_{NE}}{\theta_{EE}}\left(-\lambda_{ND}\hat{V}_D - \lambda_{NE}\hat{V}_E\right) \qquad (13) \\
\hat{R}_N &= -\frac{1}{(\sigma+\Omega)}\beta_D\left(\alpha_D \hat{V}_D + \alpha_E \hat{V}_E\right) + \frac{1}{\Delta}\left(\lambda_{ND}\hat{V}_D + \lambda_{NE}\hat{V}_E\right)
\end{aligned}
$$

The first term on the RHS of each equation in (13) represents the impact of domestic price change, while the second – the direct effect on factor rewards from factor inflows. The first effect vanishes as the elasticity of substitution tends to infinity, and we find ourselves in a standard Ricardo-Viner framework. Of primary interest is which effect will dominate if they work in opposite directions, i.e., the "global" sign of the coefficient of $\hat{V}_i$, i = D, E, N. Knowing the signs all of coefficients, we construct a table that summarises the impact left by the inflows of two types of labor on nominal factor rewards. Each cell contains two signs, representing the direct effect from factor inflow (the first sign) and the effect coming from domestic price change due to the factor inflow (the second sign). Thus two pluses (minuses) mean that factor reward definitely grows (decreases), while plus and minus mean that the effect is ambiguous.

	$\hat{R}_D$	$\hat{R}_E$	$\hat{R}_N$
	Direct (factor inflow) vs Domestic price effect	Direct (factor inflow) vs Domestic price effect	Direct (factor inflow) vs Domestic price effect
$\hat{V}_D$	- -	- +	+ -
$\hat{V}_E$	- +	- -	+ +

Table 1. Direction of change in nominal factor rewards due to different factor inflows.

A standard graphical analysis of Ricardo-Viner production process is helpful in determining the direction of change in factor rewards in ambiguous cases. We depict the inflow of both types of labor in figures 2.1. and 2.2. Higher amount of low-skilled labor in figure 2.1. moves the marginal product of capital schedule in D sector (MP(D)) upwards (from point 1 to 2), raising the output of D good, return to capital, and contracting the output of E good. However, the domestic price goes down, moving the MP(D)' curve downward to MP(D)''. If the fall in domestic price is high enough (σ small enough), the final MP(D)''' schedule will lie below the initial MP(D) curve. Equilibrium point 4 corresponds to a lower return to capital, and given constant export price the return to high-skilled labor must increase (equation 2).

We draw the effect of high-skilled labor immigration on factor prices in figure 2.2. MP(E) schedule moves upward to MP(E)', raising the return to capital and lowering the wage of low-skilled labor. The immediate increase in the domestic price raises the marginal product of capital in the domestic sector (MP(D)') and the return to high-skilled labor. If domestic price grows sufficiently (MP(D)''), the initial fall in the wage of high-skilled labor is more than outweighed.

Fig. 2.1. UNSKILLED LABOUR (D SPECIFIC FACTOR) INFLOW

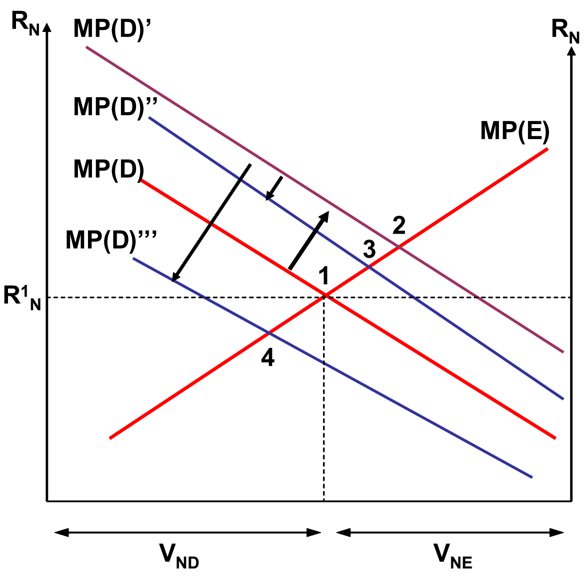
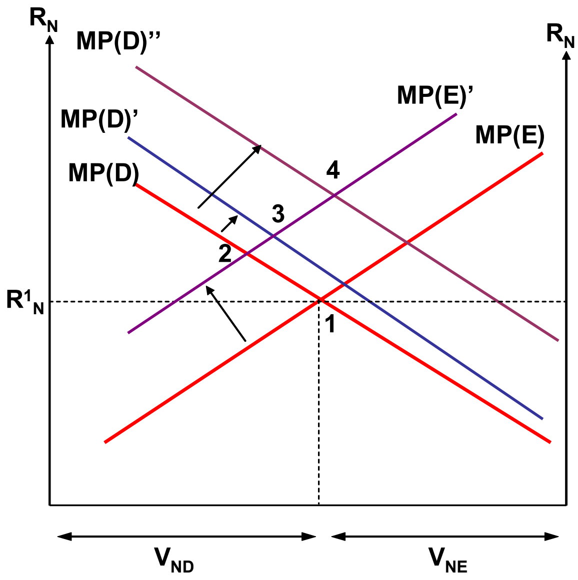


Fig. 2.2. SKILLED LABOUR (E SPECIFIC FACTOR) INFLOW



As to the critical level of sigma separating its "sufficiently" low and high values, I inspect the set of equations (13). Only ambiguous cases in the table 1 are of interest, namely, $\frac{\partial \hat{R}_E}{\partial \hat{V}_D}, \frac{\partial \hat{R}_N}{\partial \hat{V}_D}, \frac{\partial \hat{R}_D}{\partial \hat{V}_E}$. It should be noted that the "critical" value of sigma will be the same for $\frac{\partial \hat{R}_E}{\partial \hat{V}_D} \, and \, \frac{\partial \hat{R}_N}{\partial \hat{V}_D}$, since with a constant price in export sector the fall (rise) in specific factor reward implies a rise (fall) in mobile factor price.

Terms collecting and simplifying of equations in (13) yield the values of the elasticity of substitution for which the sign of the $\frac{\partial \hat{R}}{\partial \hat{V}}$ under question changes. Thus, in the case of low-skilled labor inflow the direct effect prevails $\left(\frac{\partial \hat{R}_E}{\partial \hat{V}_D} < 0, \frac{\partial \hat{R}_N}{\partial \hat{V}_D} > 0\right)$, if the elasticity of substitution between consumption goods is higher than the elasticity of factor substitution in the non-traded sector $(\sigma_D < \sigma)$, and the price effect becomes dominant $\left(\frac{\partial \hat{R}_E}{\partial \hat{V}_D} > 0, \frac{\partial \hat{R}_N}{\partial \hat{V}_D} < 0\right)$, when the elasticity of import substitution falls short of the elasticity of factor substitution in domestic sector $(\sigma_D > \sigma)$. Finally, $\hat{R}_E$ and $\hat{R}_N$ stay constant when both elasticities are equal.

The critical value of elasticity of import substitution for $\frac{\partial \hat{R}_D}{\partial \hat{V}_E}$ is $\sigma_D + \frac{\sigma_E \lambda_{NE} + \sigma_D \lambda_{ND}}{\lambda_{NE}\theta_{ND}}$, meaning that the return to domestic factor will fall (grow, not change) due to inflow of E specific factor if σ is higher (smaller, equal to) $\sigma_D + \frac{\sigma_E \lambda_{NE} + \sigma_D \lambda_{ND}}{\lambda_{NE}\theta_{ND}}$.

The following table summarizes the values for $\frac{\partial \hat{R}_i}{\partial \hat{V}_j}, i = D, E, N, j = D, N$, where the columns represent the change in low-skilled and high-skilled labor endowment, the change in factor income can be read in lines.

	$\Delta \hat{V}_D$	$\Delta \hat{V}_E$
$\Delta \hat{R}_D$	$-\frac{\lambda_{ND}}{\Delta}\frac{\theta_{ND}}{\theta_{DD}}\left(\frac{\frac{\theta_{NE}\sigma_E}{\lambda_{ND}\theta_{ND}\theta_{EE}} + \sigma}{(\sigma+\Omega)}\right)$	$\frac{\theta_{ND}}{\theta_{DD}}\frac{\lambda_{NE}}{\Delta}\left(\frac{\sigma_D + \frac{\sigma_E \lambda_{NE} + \sigma_D \lambda_{ND}}{\lambda_{NE}\theta_{ND}} - \sigma}{(\sigma+\Omega)}\right)$

$\Delta\hat{R}_E$	$\frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{ND}}{\Delta}\frac{(\sigma_D-\sigma)}{(\sigma+\Omega)}$	$-\frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{NE}}{\Delta}\left(\frac{\frac{\sigma_D}{\theta_{DD}}\left(\frac{\lambda_{ND}}{\lambda_{NE}}+\theta_{ND}\right)+\sigma}{(\sigma+\Omega)}\right)$
$\Delta\hat{R}_N$	$-\frac{\lambda_{ND}}{\Delta}\frac{(\sigma_D-\sigma)}{(\sigma+\Omega)}$	$\frac{\lambda_{NE}}{\Delta}\left(\frac{\frac{\sigma_D}{\theta_{DD}}\left(\frac{\lambda_{ND}}{\lambda_{NE}}+\theta_{ND}\right)+\sigma}{(\sigma+\Omega)}\right)$

Table 2. High-skilled and low-skilled labor inflow and factor prices.

5. POLITICAL ECONOMY OF IMMIGRATION.

We can now determine what stance nationals would take if they were to vote in a referendum for barriers to free movement of labor. The reasoning and procedure we use are similar to that of Mayer (1984), where he analyzed the process of tariff formation in a multi-sector specific factor world, assuming that individuals faced voting costs. We start with the specification of individuals' preferences and then analyze immigration policy.

We assume that preferences of citizens are homothetic and identical. The utility of individual i depends on consumption prices of domestic and import goods and her nominal income y^i:

$$U^i = U^i\left(p_D, p_M, y^i\right) \qquad (14)^{10},$$

where U^i denotes maximum utility attainable by individual i.

We will assume that individual i holds one unit of either high-skilled or low-skilled labor $V_h^i = 1$, $h = D,E$, and a positive amount of capital V_N^i. Thus, total income of individual i is:

10 An alternative way to represent individuals' utility at the *aggregate* level is given by equation (8). The utility is expressed there in terms of the consumption of a composite good and is used to determine the demand for imports and non-traded good as a function of consumption price ratio.

$$y^i = R_N V_N^i + R_h \qquad (15)$$

where R_N and R_h, $h = E, D$ denotes the returns to capital and high-skilled and low-skilled labor.

Individual i will be in favor of (against, indifferent) high-skilled or low-skilled immigrants if her utility rises (diminishes, does not change) with respective labor inflow. Following the example of Mayer (1984) on tariff formation, we take the derivative (14) with respect to low-skilled and high-skilled labor and obtain the following result:

$$\frac{\partial U^i}{\partial V_j} = \frac{\partial U^i}{\partial y^i}\left(\frac{\hat{R}_N}{\hat{V}_j}\frac{R_N}{V_j}V_N^i + \frac{\hat{R}_h}{\hat{V}_j}\frac{R_h}{V_j}V_h^i\right), \text{j = E, D , h = E, D} \qquad (16)$$

where h indicates which kind of specific labor individual i owns.

In order to determine the outcome of vote, one needs to make several assumptions about the median voter. I choose a realistic case where the immigration policy is voted for in a developed country with the following characteristics of capital distribution labor force composition. High-skilled labor is capital–rich, that is, the total amount of capital owned by high-skilled individuals is higher than the amount of capital used in the export sector: $V_E V_N^i > \lambda_{NE} V_N$, which is equivalent to $\frac{V_N^i}{V_N} > \lambda_{NE}\frac{V_E^i}{V_E}$, given that V_E^i is equal to unity. In similar terms low-skilled labor is defined as capital-poor: $\frac{V_N^i}{V_N} < \lambda_{ND}\frac{V_D^i}{V_D}$. For simplicity, there is no intra-group inequality, since all individuals are assumed to hold equal amounts of capital within each labor group.

Consider first the case where the number of high-skilled individuals exceeds that of low-skilled, meaning that median voter is high-skilled. Insertion of the respective values from table 2 into equation (16) for the high-skilled labor (h = E), and collecting terms yields:

$$\frac{\partial U_E{}^i}{\partial V_D} = \frac{\partial U_E{}^i}{\partial y^i}\left(\frac{R_N V_{ND}}{\Delta V_D}\frac{(\sigma - \sigma_D)}{(\sigma + \Omega)}\left[\frac{V_N^i}{V_N} - \lambda_{NE}\frac{V_E^i}{V_E}\right]\right)$$

$$\frac{\partial U_E{}^i}{\partial V_E} = \frac{\partial U_E{}^i}{\partial y^i}\left(\frac{R_N V_{NE}}{\Delta V_E}\left(\frac{\frac{\sigma_D}{\theta_{DD}}\left(\frac{\lambda_{ND}}{\lambda_{NE}} + \theta_{ND}\right) + \sigma}{(\sigma + \Omega)}\right)\left[\frac{V_N^i}{V_N} - \lambda_{NE}\frac{V_E^i}{V_E}\right]\right) \qquad (17)$$

Under assumption that high-skilled labor is capital-rich and given that the utility of individual i grows with her nominal income, the terms $\left[\frac{V_N^i}{V_N} - \lambda_{NE}\frac{V_E^i}{V_E}\right]$ and $\frac{\partial U_E{}^i}{\partial y^i}$ in both equations in (27) are positive, therefore, the utility of high-skilled individuals will necessarily grow with the inflow of high-skilled labor $\left(\frac{\partial U_E{}^i}{\partial V_E} > 0\right)$. As to the increase of low-skilled labor, high-skilled individuals are better (worse) off when the level of import substitution is sufficiently high (low), precisely, if the elasticity of import substitution exceeds (falls short of) elasticity of factor substitution in domestic sector ($\sigma_{HS}{*} = \sigma_D$, where $\sigma_{HS}{*}$ is the critical level of elasticity of import substitution, if median voter is high-skilled). If the two elasticities are equal, the high-skilled labor's welfare will not be affected by the growth of low-skilled labor force.

Consider next the case where the median voter is a low-skilled individual. Her attitude towards low-skilled immigrants can be expressed as follows:

$$\frac{\partial U_D{}^i}{\partial V_D} = -\frac{\partial U_D{}^i}{\partial y^i}\left(\frac{R_N V_{ND}\sigma\left(\frac{V_D^i}{V_D}\lambda_{ND} - \frac{V_N^i}{V_N}\right) + \mu}{\Delta(\sigma + \Omega)V_D}\right) \qquad (18),$$

where $\mu = R_N \left(V_{NE} \frac{\sigma_E}{\theta_{EE}} \frac{V_D^i}{V_D} + \sigma_D \lambda_{ND} V_N^i \right) > 0$.

Given that low-skilled labor is capital poor, i.e. $\frac{V_N^i}{V_N} < \lambda_{ND} \frac{V_D^i}{V_D}$, the RHS of (18) is negative, meaning that low-skilled individuals are always affected adversely by the inflow of low-skilled immigrants.

Regarding high-skilled immigration, the low-skilled individuals will take a negative stance towards high-skilled individuals only at sufficiently high levels of elasticity of import demand. It can be shown from equation that the critical level of sigma $\sigma_{LS}{}^{*}$ is finite, positive, and necessarily higher than σ_D, and decreases in the degree of inequality of capital distribution[11]. In the limiting case, where the low-skilled individuals own all the capital used in the non-traded domestic sector, the critical level of elasticity of import demand tends to infinity, implying a positive attitude towards high-skilled migrants.

Thus, a country characterized by a low-skilled median voter will be opposed to the inflow of unskilled labor and will favor (oppose) high-skilled immigration if the elasticity of import demand is sufficiently low (high).

Table 3 summarizes the attitudes of the native population towards high- and low-skilled immigrants depending on the skill level of the median voter and the value of elasticity of substitution between imports and the non-traded good. In a country with important non-traded sector (when domestic and imported goods are poor substitutes), the nationals, independently of their skill level, will always favour the inflow of high-skilled labor and oppose low-skilled immigrants. This result might explain the preference for high-skilled immigration in developed economies, assuming that low-skilled labor is concentrated in non-

[11] The exact value for the critical level of elasticity of demand for imports is given by

$$\sigma_{LS}{}^{*} = \frac{y_D^i \frac{\sigma_D}{\theta_{DD}} \left(\theta_{ND} \lambda_{NE} + \lambda_{ND} \right) + R_D V_D^i \frac{\sigma_E \lambda_{NE}}{\theta_{DD}}}{R_N V_{NE} \left(\frac{V_D^i}{V_D} \lambda_{ND} - \frac{V_N^i}{V_N} \right)}.$$

traded sectors. However, for high levels of elasticity of substitution (including the limiting case of perfect substitution, or the absence of non-traded sector) , the skill level of the median voter is crucial in the determination of nationals' overall stance towards *all* types of immigrants: a high- (low-) skilled median voter will be pro- (anti-) immigrant.

		Median voter	
		High-skilled	Low-skilled
Elasticity of substitution between imported and non-traded goods	$\sigma < \sigma_{HS}*(=\sigma_D)$	For high-skilled Against low-skilled	For high-skilled Against low-skilled
	$\sigma_{HS}* < \sigma < \sigma_{LS}*$	For high-skilled For low-skilled	For high-skilled Against low-skilled
	$\sigma_{LS}* < \sigma$	For high-skilled For low-skilled	Against high-skilled Against low-skilled

Table 3. Attitudes towards immigration.

In broad terms, the results in table 2 suggest that countries with the majority represented by high-skilled (low-skilled) workers should adopt more positive (negative) attitudes towards immigration. To compare this finding with actual European evidence, one needs to establish the skill level of the median voter in each country. I will assume that in European economies the proportion of the high-skilled is proportional to the relative wealth of the country. The labor productivity of qualified labor force is usually higher than that of the low-skilled, meaning that higher income per capita should be associated with relatively abundant high-skilled labor. Turning back to graph 1 in section 1 of the paper, one can deduce a negative relationship between per capita income and anti-immigration feelings in Europe[12], suggesting that higher skill level of voters makes them more pro-immigrant, and thus confirming the theoretical result mentioned above.

[12] Because of the problem known as "hypothetical bias", results from opinion polls do not necessarily reflect what citizens would actually vote for. See de Melo, Miguet and Mueller (2002).

6. CONCLUSIONS

A number of social surveys conducted in developed western economies indicate that attitudes toward immigration are mostly negative. However, as shown in the European opinion polls, attitudes vary with per-capita income of citizens, and feelings about foreign labor with different skill level are not the same. As a rule, high-skilled labor is welcome, while a much more antagonistic stance is adopted toward the immigration of low-skilled workers. At the same time, in most developed economies high-skilled labor is concentrated in export industries and low-skilled labor is specific to non-traded sectors. Labor is becoming increasingly immobile between industries. I embody these features into a general equilibrium model with specific factors and a non-traded good to analyse the impact of high-skilled and low-skilled labor inflow on domestic factor prices. I assume that immigrants are sector-specific and do not change the stock of capital of the economy. The utility of each individual depends on her income and consumption prices, and the direction of change in utility determines what stance a person will take when asked to vote for high-skilled and low-skilled immigration. The political economy outcome is determined by majority rule.

I conclude that the degree of substitution between non-traded and imported goods and the skill level of the median are crucial in the referendum outcome. If imported and domestic non-traded goods are sufficiently poor substitutes, national voters regardless of their skill level will be against low-skilled foreign labor and will favour the inflow of highly qualified individuals. Assuming that domestic and foreign low-skilled labor is specific to non-traded sectors, e.g., tourism services and construction, this result can explain relatively more antagonistic feelings about unskilled labor in developed economies and immigration policies conditioned on immigrants' skill level.

If the non-traded and imported goods are strong substitutes, the skill level of the median voter is decisive in the determining country's stance towards immigrants. In this case a high-skilled median voter will favor both types if foreign labor, while a low-skilled majority will oppose immigrants regardless of their skill level.

On the economy-wide level, I find that voters with higher skill level will be more tolerant towards all types of immigrants, a result explaining a more antagonistic stance towards immigrants in the low per-capita income countries of Southern Europe, and more tolerant feelings in the rich European economies, such as Scandinavian countries, Switzerland and Luxembourg.

Several extensions of the model developed in this paper are possible. First, it is suited for the analysis of welfare change in a *sending* country with a non-traded sector. Although *emigration* is rarely a voting issue (emigrants usually are not confronted with restrictions to leave their home country), our framework can answer how factor prices and individuals' utility will alter with the outflow of high-skilled (brain-drain) and low-skilled labor. The issue is particularly important regarding the gradual introduction of free movement of labor in the enlarged Europe and the impact of emigration on Eastern European countries. Assuming again that the high-skilled (low-skilled) individuals are capital-rich (-poor) and that low-skilled labor is employed in the sector producing a non-traded good, the changes in individuals' welfare in the sending country are exactly opposite to those in table 3. At low levels of elasticity of substitution between non-traded and imported goods, the voters (both low-skilled and high-skilled) in the sending country are better off in real income terms with the emigration of low-skilled labor and worse off, if highly qualified labor flows out. This result is important in explaining the recent and possible future EU negotiation processes over enlargement and gradual introduction of free migration[13]. Given that both in old and candidate states there are non-traded sectors, and considering the case of *non-skilled* labor movements across borders as major issue of negotiations[14], voters in sending applicant countries are in favor of low-skilled *e*migration, i.e., the immediate removal of barriers to labor flows, while voters in old member states are against low-skilled *im*migration.

The second extension is the introduction of the international capital movements (with the price of capital given by the world market), which would permit to explore the link between

[13] During the negotiations over free migration in the EU eastward enlargement process, old member states where opposed to the immediate removal of immigration barriers, while the candidate states favoured it. This resulted in the introduction of transition periods.

[14] The movements of low-skilled labor was one the major concerns in the negotiations over the EU eastward enlargement. The potential high-skilled immigrants were concerned to a much lesser extent, since much of the potential high-skilled immigration had occurred before the adhesion of the new states, and because of the fact that high-skilled individuals have less incentives to move due to higher income in their home countries.

migration and FDI. The issue is particularly important regarding the impact of the increased outflow of labor on FDI in the Eastern European economies.

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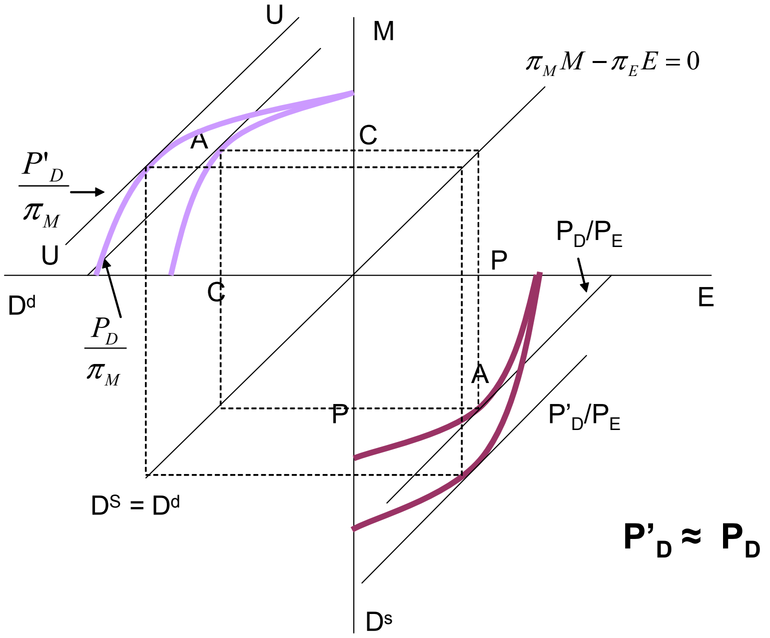
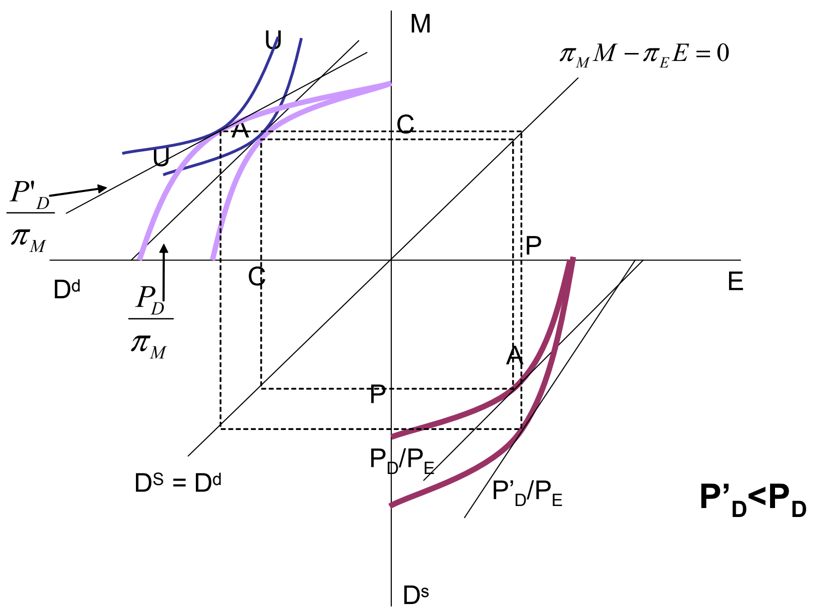
Appendix I.

Factor inflows under high and low levels of sigma.

Increase of low-skilled labour (sigma high)



Increase of low-skilled labour (sigma low)



Appendix II

A.1. Derivation of (1) and (2)

Under the assumption of full employment of factors the following conditions hold:

$$a_{DD}D = V_D \qquad \text{(A.1)}$$

$$a_{EE}E = V_E \qquad \text{(A.2)}$$

$$a_{ND}D + a_{NE}E = V_N \qquad \text{(A.3)}$$

The competitive profit relations state that in equilibrium unit costs will equal market price (unit cost equations):

$$a_{DD}R_D + a_{ND}R_N = p_D \qquad \text{(A.4)}$$

$$a_{EE}R_E + a_{NE}R_N = p_E\ , \qquad \text{(A.5)}$$

From (A.1), (A.2), (A.3) we obtain:

$$\frac{a_{ND}}{a_{DD}}V_D + \frac{a_{NE}}{a_{EE}}V_E = V_N \qquad \text{(A.6)}$$

Differentiating the unit cost equations yields:

$$\partial a_{DD}R_D + a_{DD}\partial R_D + \partial a_{ND}R_N + a_{ND}\partial R_N = \partial p_D$$

$$\frac{a_{DD}\partial R_D R_D}{R_D} + \frac{a_{ND}\partial R_N R_N}{R_N} = \frac{\partial p_D p_D}{p_D} - \left(\frac{\partial a_{DD}R_D a_{DD}}{a_{DD}} + \frac{\partial a_{ND}R_N a_{ND}}{a_{ND}}\right)$$

$$\frac{a_{DD}\hat{R}_D R_D}{p_D} + \frac{a_{ND}\hat{R}_N R_N}{p_D} = \hat{p}_D - \left(\frac{\hat{a}_{DD}R_D a_{DD}}{p_D} + \frac{\hat{a}_{ND}R_N a_{ND}}{p_D}\right)$$

$$\frac{V_D R_D}{Dp_D}\hat{R}_D + \frac{V_{ND}R_N}{Dp_D}\hat{R}_N = \hat{p}_D - \left(\frac{V_D R_D}{Dp_D}\hat{a}_{DD} + \frac{V_{ND}R_N}{Dp_D}\hat{a}_{ND}\right)$$

$\theta_{DD}\hat{R}_D + \theta_{ND}\hat{R}_N = \hat{p}_D$, where the use was made by the envelope theorem $\left(\theta_{DD}\hat{a}_{DD} + \theta_{ND}\hat{a}_{ND} = 0\right)$

By permuting the subscripts we obtain equation (2).

A.2. Derivation of (4) and (5).

Totally differentiate (A.6):

$$\left(\frac{da_{ND}a_{DD}-da_{DD}a_{ND}}{(a_{DD})^2}\right)V_D+\frac{a_{ND}}{a_{DD}}dV_D+\left(\frac{da_{NE}a_{EE}-da_{EE}a_{NE}}{(a_{EE})^2}\right)V_E+\frac{a_{NE}}{a_{EE}}dV_E=dV_N$$

$$\left(\frac{da_{ND}}{a_{DD}}-\hat{a}_{DD}\frac{a_{ND}}{a_{DD}}\right)V_D+\frac{a_{ND}}{a_{DD}}dV_D+\left(\frac{da_{NE}}{a_{EE}}-\hat{a}_{EE}\frac{a_{NE}}{a_{EE}}\right)V_E+\frac{a_{NE}}{a_{EE}}dV_E=dV_N$$

$$((\hat{a}_{ND}-\hat{a}_{DD})V_D+dV_D)\frac{a_{ND}}{a_{DD}}+((\hat{a}_{NE}-\hat{a}_{EE})V_E+dV_E)\frac{a_{NE}}{a_{EE}}=dV_N$$

$$((\hat{a}_{ND}-\hat{a}_{DD})+\hat{V}_D)\frac{V_Da_{ND}}{a_{DD}V_N}+((\hat{a}_{NE}-\hat{a}_{EE})+\hat{V}_E)\frac{V_Ea_{NE}}{a_{EE}V_N}=\hat{V}_N$$

$$((\hat{a}_{ND}-\hat{a}_{DD})+\hat{V}_D)\frac{Da_{ND}}{V_N}+((\hat{a}_{NE}-\hat{a}_{EE})+\hat{V}_E)\frac{Ea_{NE}}{V_N}=\hat{V}_N$$

Using the definitions of σ_D, σ_E *and* λ:

$$(\sigma_D(\hat{R}_D-\hat{R}_N)+\hat{V}_D)\lambda_{ND}+(\sigma_E(\hat{R}_E-\hat{R}_N)+\hat{V}_E)\lambda_{NE}=\hat{V}_N$$

$$\hat{R}_D\sigma_D\lambda_{ND}+\hat{R}_E\sigma_E\lambda_{NE}-\hat{R}_N(\sigma_D\lambda_{ND}+\sigma_E\lambda_{NE})=\hat{V}_N-\lambda_{ND}\hat{V}_D-\lambda_{NE}\hat{V}_E \qquad \text{(A.7)}$$

Use (1) and (2):

$$\hat{R}_N=\frac{\hat{p}_D-\theta_{DD}\hat{R}_D}{\theta_{ND}}=\frac{\hat{p}_E-\theta_{EE}\hat{R}_E}{\theta_{NE}}$$

$$\theta_{NE}\hat{p}_D-\theta_{DD}\hat{R}_D\theta_{NE}=\theta_{ND}\hat{p}_E-\theta_{EE}\hat{R}_E\theta_{ND}$$

$$\hat{R}_E=\frac{-\theta_{NE}\hat{p}_D+\theta_{DD}\hat{R}_D\theta_{NE}+\theta_{ND}\hat{p}_E}{\theta_{EE}\theta_{ND}}$$

And insert into (A.7) and multiply both sides by $\frac{\theta_{ND}}{\theta_{DD}}$:

$$\hat{R}_D\left[\sigma_D\lambda_{ND}\frac{\theta_{ND}}{\theta_{DD}}+\frac{\theta_{NE}}{\theta_{EE}}\sigma_E\lambda_{NE}+(\sigma_D\lambda_{ND}+\sigma_E\lambda_{NE})\right]+\hat{p}_D\left[-\frac{\theta_{NE}}{\theta_{EE}\theta_{DD}}\sigma_E\lambda_{NE}-\frac{1}{\theta_{DD}}(\sigma_D\lambda_{ND}+\sigma_E\lambda_{NE})\right]+$$
$$+\hat{p}_E\left[\frac{1}{\theta_{EE}}\frac{\theta_{ND}}{\theta_{DD}}\sigma_E\lambda_{NE}\right]=\left[\hat{V}_N-\lambda_{ND}\hat{V}_D-\lambda_{NE}\hat{V}_E\right]\frac{\theta_{ND}}{\theta_{DD}}$$

$$\hat{R}_D\left[\frac{\sigma_D\lambda_{ND}}{\theta_{DD}}+\frac{\sigma_E\lambda_{NE}}{\theta_{EE}}\right]+\hat{p}_D\left[-\frac{1}{\theta_{DD}}\sigma_D\lambda_{ND}-\sigma_E\lambda_{NE}\frac{1}{\theta_{EE}\theta_{DD}}\right]++\hat{p}_E\left[\frac{1}{\theta_{EE}}\frac{\theta_{ND}}{\theta_{DD}}\sigma_E\lambda_{NE}\right]$$
$$=\left[\hat{V}_N-\lambda_{ND}\hat{V}_D-\lambda_{NE}\hat{V}_E\right]\frac{\theta_{ND}}{\theta_{DD}}$$

Define $\Delta=\sum_{i=D,E}\lambda_{Ni}\frac{\sigma_i}{\theta_{ii}};\quad \beta_{i,i=D,E}=\frac{\lambda_{Ni}\frac{\sigma_i}{\theta_{ii}}}{\sum_i\lambda_{Ni}\frac{\sigma_i}{\theta_{ii}}}$

$$\hat{R}_D=\left[\beta_D+\frac{1}{\theta_{DD}}\beta_E\right]\hat{p}_D-\frac{\theta_{ND}}{\theta_{DD}}\beta_E\hat{p}_E+\frac{1\,\theta_{ND}}{\Delta\theta_{DD}}\left(\hat{V}_N-\lambda_{ND}\hat{V}_D-\lambda_{NE}\hat{V}_E\right)\qquad(5)$$

Derivation of equation (5):

From (1):

$$\hat{R}_D=\frac{\hat{p}_D-\theta_{ND}\hat{R}_N}{\theta_{DD}}$$

Insert it into (4) and collect terms:

$$\frac{\hat{p}_D-\theta_{ND}\hat{R}_N}{\theta_{DD}}=\left[\beta_D+\frac{1}{\theta_{DD}}\beta_E\right]\hat{p}_D-\frac{\theta_{ND}}{\theta_{DD}}\beta_E\hat{p}_E+\frac{1\,\theta_{ND}}{\Delta\theta_{DD}}\left(\hat{V}_N-\lambda_{ND}\hat{V}_D-\lambda_{NE}\hat{V}_E\right)$$

$$\hat{R}_N=\beta_D\hat{p}_D+\beta_E\hat{p}_E+\frac{1}{\Delta}\left(\lambda_{ND}\hat{V}_D+\lambda_{NE}\hat{V}_E-\hat{V}_N\right)$$

A.3 Derivation of (7)

Use (3) and the envelope theorem $\left(\theta_{ii}\hat{a}_{ii}+\theta_{Ni}\hat{a}_{Ni}=0,\ i=D,E\right)$:

$$\sigma_D = \frac{(\hat{a}_{ND} - \hat{a}_{DD})}{(\hat{R}_D - \hat{R}_N)}$$

$$\hat{a}_{ND} = \sigma_D(\hat{R}_D - \hat{R}_N) + \hat{a}_{DD}$$

$$\theta_{DD}\hat{a}_{DD} + \theta_{ND}(\sigma_D(\hat{R}_D - \hat{R}_N) + \hat{a}_{DD}) = 0$$

$$\hat{a}_{DD}(\theta_{DD} + \theta_{ND}) = -\theta_{ND}\sigma_D(\hat{R}_D - \hat{R}_N)$$

$$\hat{a}_{DD} = -\theta_{ND}\sigma_D(\hat{R}_D - \hat{R}_N) \qquad \text{(A.8)}$$

Use (A.1), (A.2), (A.8), (4) and (5):

$$\hat{D} - \hat{E} = (\hat{V}_D - \hat{V}_E) + (\hat{a}_{EE} - \hat{a}_{DD})$$

$$\hat{D} - \hat{E} = (\hat{V}_D - \hat{V}_E) + (-\theta_{NE}\sigma_E(\hat{R}_E - \hat{R}_N) + \theta_{ND}\sigma_D(\hat{R}_D - \hat{R}_N))$$

$$(\hat{D} - \hat{E}) = \left(\theta_{ND}\frac{\sigma_D}{\theta_{DD}}\beta_E + \theta_{NE}\frac{\sigma_E}{\theta_{EE}}\beta_D\right)(\hat{p}_D - \hat{p}_E) +$$

$$+ \left(\hat{V}_D - \hat{V}_E\right) + \frac{1}{\Delta}\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)\left(\hat{V}_N - \lambda_{ND}\hat{V}_D - \lambda_{NE}\hat{V}_E\right)$$

A.4 to show that $\frac{M}{D} = k\left(\frac{p_D}{p_M}\right)^{\sigma}$ (eq. 9)

Define $(\sigma - 1)/\sigma = -\rho$ in (8) and derive $Q(.)$ with respect to D and M :

$$Q(M, D, \sigma) = \left[\chi M^{-\rho} + (1-\chi)D^{-\rho}\right]^{-1/\rho}$$

$$\frac{dQ}{dD} = (-\frac{1}{\rho})\left[\chi M^{-\rho} + (1-\chi)D^{-\rho}\right]^{-(1/\rho)-1}(1-\chi)(-\rho)D^{-\rho-1}$$

$$\frac{dQ}{dD} = (1-\chi)\left[\chi M^{-\rho} + (1-\chi)D^{-\rho}\right]^{-(\frac{1+\rho}{\rho})}D^{-(1+\rho)}$$

$$\frac{dQ}{dD} = (1-\chi)(\frac{Q}{D})^{1+\rho}$$

Similarly obtain the expression for $\frac{dQ}{dM}$:

$$\frac{dQ}{dM} = \chi(\frac{Q}{M})^{1+\rho}$$

The slope of the consumption indifference curve is equal to the negative ratio of marginal utilities from both products :

$$\frac{\partial M}{\partial D} = -\frac{dQ/dD}{dQ/dM} = -\frac{(1-\beta)}{\beta}(\frac{M}{D})^{1+\rho}$$

At the optimum the slope of the indifference curve is equal to the negative price ratio:

$$\frac{\partial M}{\partial D} = -\frac{p_D}{p_M}$$

$$\frac{(1-\chi)}{\chi}(\frac{M}{D})^{1+\rho} = \frac{p_D}{p_M}$$

$$\frac{M}{D} = \left[\frac{\chi}{(1-\chi)}(\frac{p_D}{p_M})\right]^{\frac{1}{1+\rho}} = \frac{\chi}{(1-\chi)}^{\frac{1}{1+\rho}}\left(\frac{p_D}{p_M}\right)^{\frac{1}{1+\rho}} = \left(\frac{\beta}{(1-\beta)}\right)^{\sigma}\left(\frac{p_D}{p_M}\right)^{\sigma} = k(\frac{p_D}{p_M})^{\sigma}$$

A.5. Determination of the sign of coefficients $\alpha_i, i = D, E$ in (12)

To prove that $\alpha_D = 1 - \lambda_{ND}\frac{1}{\Delta}\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)$ is positive, consider the second term of the RHS:

$$\lambda_{ND}\frac{1}{\Delta}\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right) =$$

$$\frac{\lambda_{ND}\theta_{DD}\theta_{EE}}{(\lambda_{ND}\sigma_D\theta_{EE}+\lambda_{NE}\sigma_D\theta_{DD})}\frac{(\theta_{ND}\sigma_D\theta_{EE}-\theta_{NE}\sigma_E\theta_{DD})}{\theta_{DD}\theta_{EE}} = \frac{\theta_{ND}\left(\sigma_D\theta_{EE} - \frac{\theta_{NE}}{\theta_{ND}}\sigma_E\theta_{DD}\right)}{\left(\sigma_D\theta_{EE}+\frac{\lambda_{NE}}{\lambda_{ND}}\sigma_E\theta_{DD}\right)}$$

With all parameters defined positive this term can never be higher than one. Subtracting it from 1 yields some positive value.

Similarly, $\alpha_E = -1 - \lambda_{NE}\frac{1}{\Delta}\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)$ is necessarily negative.

A.6.1. Derivation of $\frac{\partial \hat{R}_E}{\partial \hat{V}_D}, \frac{\partial \hat{R}_N}{\partial \hat{V}_D}$ **and** $\frac{\partial \hat{R}_D}{\partial \hat{V}_E}$ **("ambiguous" cases) in (13)**

$$\frac{\partial \hat{R}_E}{\partial \hat{V}_D} = \frac{1}{(\sigma+\Omega)}\frac{\theta_{NE}}{\theta_{EE}}\beta_D\alpha_D - \lambda_{ND}\frac{1}{\Delta}\frac{\theta_{NE}}{\theta_{EE}} = \frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{ND}}{\Delta}\frac{1}{(\sigma+\Omega)}\left(\frac{\sigma_D}{\theta_{DD}}\alpha_D - (\sigma+\Omega)\right) =$$

$$= \frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{ND}}{\Delta}\frac{1}{(\sigma+\Omega)}\left(\frac{\sigma_D}{\theta_{DD}} - \beta_D\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right) - \sigma - \left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}}\beta_E + \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\beta_D\right)\right) =$$

$$= \frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{ND}}{\Delta}\frac{1}{(\sigma+\Omega)}\left(\frac{\sigma_D}{\theta_{DD}} - \frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \sigma\right) = \frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{ND}}{\Delta}\frac{(\sigma_D - \sigma)}{(\sigma+\Omega)}$$

$$\frac{\partial \hat{R}_N}{\partial \hat{V}_D} = -\frac{1}{(\sigma+\Omega)}\beta_D\alpha_D + \frac{1}{\Delta}\lambda_{ND} = -\frac{\lambda_{ND}}{\Delta}\frac{(\sigma_D - \sigma)}{(\sigma+\Omega)}$$

$$\frac{\partial \hat{R}_D}{\partial \hat{V}_E} = -\frac{1}{(\sigma+\Omega)}\left[\beta_D + \frac{1}{\theta_{DD}}\beta_E\right]\alpha_E - \lambda_{NE}\frac{1}{\Delta}\frac{\theta_{ND}}{\theta_{DD}} =$$

$$= \frac{\theta_{ND}}{\theta_{DD}}\frac{\lambda_{NE}}{\Delta}\frac{1}{(\sigma+\Omega)}\left[\left(\beta_D + \frac{1}{\theta_{DD}}\beta_E\right)\left(\frac{\Delta}{\lambda_{NE}}\frac{\theta_{DD}}{\theta_{ND}} + \frac{\theta_{DD}}{\theta_{ND}}\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)\right) - (\sigma+\Omega)\right] =$$

$$= \frac{\theta_{ND}}{\theta_{DD}}\frac{\lambda_{NE}}{\Delta}\frac{1}{(\sigma+\Omega)}\left[\frac{\lambda_{ND}}{\lambda_{NE}}\frac{\sigma_D}{\theta_{ND}} + \frac{\sigma_E}{\theta_{EE}\theta_{ND}} + \sigma_D - \frac{\theta_{NE}\sigma_E}{\theta_{ND}\theta_{EE}} - \sigma\right] = \frac{\theta_{ND}}{\theta_{DD}}\frac{\lambda_{NE}}{\Delta}\left(\frac{\sigma_D + \frac{\sigma_E\lambda_{NE} + \sigma_D\lambda_{ND}}{\lambda_{NE}\theta_{ND}} - \sigma}{(\sigma+\Omega)}\right)$$

A.6.2. Derivation of $\frac{\partial \hat{R}_D}{\partial \hat{V}_D}, \frac{\partial \hat{R}_E}{\partial \hat{V}_E}, \frac{\partial \hat{R}_N}{\partial \hat{V}_E}$ **("unambiguous" cases) in (13)**

$$\frac{\partial \hat{R}_D}{\partial \hat{V}_D} = -\frac{1}{(\sigma+\Omega)}\left[\beta_D + \frac{1}{\theta_{DD}}\beta_E\right]\alpha_D - \lambda_{ND}\frac{1\,\theta_{ND}}{\Delta\theta_{DD}} =$$

$$= -\frac{\theta_{ND}}{\theta_{DD}}\frac{\lambda_{ND}}{\Delta}\frac{1}{(\sigma+\Omega)}\left[\left(\beta_D + \frac{1}{\theta_{DD}}\beta_E\right)\left(\frac{\Delta}{\lambda_{ND}}\frac{\theta_{DD}}{\theta_{ND}} - \frac{\theta_{DD}}{\theta_{ND}}\left(\frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right)\right) + (\sigma+\Omega)\right] =$$

$$= -\frac{\lambda_{ND}}{\Delta}\frac{\theta_{ND}}{\theta_{DD}}\frac{1}{(\sigma+\Omega)}\left[\frac{\lambda_{NE}\theta_{NE}\sigma_E}{\lambda_{ND}\theta_{ND}\theta_{EE}} + \frac{\theta_{NE}\sigma_E}{\theta_{ND}\theta_{EE}} + \sigma\right] = -\frac{\lambda_{ND}}{\Delta}\frac{\theta_{ND}}{\theta_{DD}}\left(\frac{\frac{\theta_{NE}\sigma_E}{\lambda_{ND}\theta_{ND}\theta_{EE}} + \sigma}{(\sigma+\Omega)}\right)$$

$$\frac{\partial \hat{R}_E}{\partial \hat{V}_E} = \frac{1}{(\sigma+\Omega)}\frac{\theta_{NE}}{\theta_{EE}}\beta_D\alpha_E - \lambda_{NE}\frac{1\,\theta_{NE}}{\Delta\theta_{EE}}$$

$$= -\frac{1}{(\sigma+\Omega)}\frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{NE}}{\Delta}\left(\beta_D\left(\frac{\Delta}{\lambda_{NE}} + \frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right) + (\sigma+\Omega)\right) =$$

$$= -\frac{1}{(\sigma+\Omega)}\frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{NE}}{\Delta}\left(\frac{\lambda_{ND}\sigma_D}{\lambda_{NE}\theta_{DD}} + \frac{\theta_{ND}\sigma_D}{\theta_{DD}} + \sigma\right) = -\frac{\theta_{NE}}{\theta_{EE}}\frac{\lambda_{NE}}{\Delta}\left(\frac{\frac{\sigma_D}{\theta_{DD}}\left(\frac{\lambda_{ND}}{\lambda_{NE}} + \theta_{ND}\right) + \sigma}{(\sigma+\Omega)}\right)$$

$$\frac{\partial \hat{R}_E}{\partial \hat{V}_E} = -\frac{1}{(\sigma+\Omega)}\frac{\theta_{NE}}{\theta_{EE}}\alpha_E + \lambda_{NE}\frac{1}{\Delta} =$$

$$= \frac{1}{(\sigma+\Omega)}\frac{\lambda_{NE}}{\Delta}\left(\beta_D\left(\frac{\Delta}{\lambda_{NE}} + \frac{\theta_{ND}\sigma_D}{\theta_{DD}} - \frac{\theta_{NE}\sigma_E}{\theta_{EE}}\right) + (\sigma+\Omega)\right) =$$

$$= \frac{1}{(\sigma+\Omega)}\frac{\lambda_{NE}}{\Delta}\left(\frac{\lambda_{ND}\sigma_D}{\lambda_{NE}\theta_{DD}} + \frac{\theta_{ND}\sigma_D}{\theta_{DD}} + \sigma\right) = \frac{\lambda_{NE}}{\Delta}\left(\frac{\frac{\sigma_D}{\theta_{DD}}\left(\frac{\lambda_{ND}}{\lambda_{NE}} + \theta_{ND}\right) + \sigma}{(\sigma+\Omega)}\right)$$

A.7. Derivation of (16)

Differentiate (14) with respect to V_j:

$$\frac{\partial U^i}{\partial V_j} = \frac{\partial U^i}{\partial p_M}\frac{\partial p_M}{\partial V_j} + \frac{\partial U^i}{\partial p_D}\frac{\partial p_D}{\partial V_j} + \frac{\partial U^i}{\partial y_i}\frac{\partial y_i}{\partial V_j}$$

Let ϕ_i be the share of individual i's income in aggregate output of the economy Y, which is equal to the sum of all factor incomes:

$$\phi^i = \frac{\left(R_N V_N^i + R_h\right)}{\left(R_N V_N + \sum_{j=E,D} R_j V_j\right)}$$

Roy's identity and the property of homothetic utility functions that the *i*th person's demand for a commodity equals the product of its income share and aggregate demand for the same commodity, yield:

$$\frac{\partial U^i}{\partial V_j} = \frac{\partial U^i}{\partial y^i}\left(-\phi^i M \frac{\partial p_M}{\partial V_j} - \phi^i D \frac{\partial p_D}{\partial V_j} + Y \frac{\partial \phi^i}{\partial V_j} + \phi^i \frac{\partial Y}{\partial V_j}\right) =$$

$$= \frac{\partial U^i}{\partial y^i}\left[-\phi^i M \frac{\partial p_M}{\partial V_j} - \phi^i D \frac{\partial p_D}{\partial V_j} + Y \frac{\partial \phi^i}{\partial V_j} + \phi^i \left(\frac{\partial p_D}{\partial V_j} D + p_D \frac{\partial D}{\partial V_j} + \frac{\partial p_E}{\partial V_j} E + p_E \frac{\partial E}{\partial V_j}\right)\right]$$

Making use of the fact that world import and export prices are exogenous $\left(\frac{\partial p_M}{\partial V_j} = \frac{\partial p_E}{\partial V_j} = 0\right)$, and of the property of GDP function $w_j = \sum_1^n p_i \frac{\partial Q_i}{\partial v_j}$ (Wong, p.47, eq. 2.39b), the expression for $\frac{\partial U^i}{\partial V_D}$ reduces to :

$$\frac{\partial U^i}{\partial V_j} = \frac{\partial U^i}{\partial y^i}\left(-\phi^i M \frac{\partial p_M}{\partial V_j} - \phi^i D \frac{\partial p_D}{\partial V_j} + Y \frac{\partial \phi^i}{\partial V_j} + \phi^i \frac{\partial Y}{\partial V_j} \right) =$$

$$= \frac{\partial U^i}{\partial y^i}\left[-\phi^i D \frac{\partial p_D}{\partial V_j} + Y \frac{\partial \phi^i}{\partial V_j} + \phi^i \left(\frac{\partial p_D}{\partial V_j} D + p_D \frac{\partial D}{\partial V_j} + \frac{\partial p_E}{\partial V_j} E + p_E \frac{\partial E}{\partial V_j} \right)\right]$$

$$= \frac{\partial U^i}{\partial y^i}\left(Y \frac{\partial \phi^i}{\partial V_j} + \phi^i R_j \right) = \frac{\partial U^i}{\partial y^i}\left(Y \left[\frac{\left(\frac{\partial R_N}{\partial V_j} V_N^i + \frac{\partial R_h}{\partial V_j} V_h^i \right) - \phi^i \left(R_j + \sum_{i=N,E,D} \frac{\partial R_i}{\partial V_j} V_i \right)}{Y} \right] + \phi^i R_j \right)$$

Keeping in mind that $\sum_{l=1}^{m} v_l \frac{\partial w_l}{\partial v_j} = 0$ (Wong, p.46, eq. 2.37), one gets the final expression for the change in utility:

$$\frac{\partial U^i}{\partial V_j} = \frac{\partial U^i}{\partial y^i}\left(\frac{\partial R_N}{\partial V_j} V_N^i + \frac{\partial R_h}{\partial V_j} V_h^i \right) = \frac{\partial U^i}{\partial y^i}\left(\frac{\hat{R}_N}{\hat{V}_j} \frac{R_N}{V_j} V_N^i + \frac{\hat{R}_h}{\hat{V}_j} \frac{R_h}{V_j} V_h^i \right)$$